Study and Analysis of a New Five-Dimensional Hyper-Chaotic System

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ABSTRACT

This study presents a novel Five-Dimensional (5D) hyper-chaotic system to increase the degree of disorder in a system, which comprises 10 positive chaotic parameters as well as complex chaotic dynamic properties. The system's basic properties and dynamic behavior are investigated based on the existence of equilibrium points, Lyapunov Exponents (LEs), chaotic attractors, dissipative properties, symmetry, waveform analysis, Kaplan-Yorke dimensions, bifurcation properties, and sensitivity to initial conditions. The new system has 5 LEs in addition to 2 points of unstable equilibrium. According to the study's findings, the Maximal positive Lyapunov Exponent (MLE) and Kaplan-Yorke estimated values are 6.85408 and 4.37292, respectively. The results show that the innovative system exhibits highly complicated, unstable, and unpredictably unstable properties.

Keywords-hyper-chaotic; five-dimensional; sensitivity to initial conditions; unstable; Lyapunov exponent; waveform analysis; bifurcation

I. INTRODUCTION

Chaos can be implemented using a nonlinear dynamic system. It is proved that chaos is deterministic, periodic, and response sensitive to small changes, control parameters, and initial conditions [1-3]. Hyper chaos was first reported in [4], and its initial implementation was achieved in [5], using analog circuits. Over the last years, researchers have extensively studied hyper-chaotic systems because they exhibit at least two positive LEs. Hyper-chaotic systems are commonly preferred over the chaotic system because their dynamics have been expanded in many directions and can generate more intricate attractors [6, 7]. On the other hand, the chaotic system has only one positive LE [8].

Many researchers studied the 5D chaotic system over the last few years [9-16]. In [9], the chaotic sequence generated by the hyper-chaotic system was utilized as a key to address the main issue in improving the Advanced Encryption Standard (AES) without altering the core cryptographic process used in the original AES. The LEs were determined as: Ly_1 = 0.315207, Ly_2 = 0.135137, Ly_3 = −0.082011, Ly_4 = −0.275085, and Ly_5 = −11.9927. Authors in [10] used the flux controlled memristor to develop a 5D chaotic system. The examination of this system's dynamics revealed its hyper-chaotic properties. The study aimed to facilitate the analysis and development of synchronization, for both the new chaotic memristor system and its corresponding slave system. The LEs were determined as: Ly_1 = 1.0241, Ly_2 = 0.0137, Ly_3 = −0.1735, Ly_4 = −2.3787, and Ly_5 = −70.3244.

Authors in [11] proposed a 5D hyper-chaotic system showcasing its dynamic features, like phase diagrams, LEs, time series analysis, equilibrium points, and bifurcation diagrams. The LEs were determined as: Ly_1 = 0.62481, Ly_2 = 0.099614, Ly_3 = −0.0030413, Ly_4 = −12.6821, and Ly_5 = −19.6883. In [12], a 5D chaotic system was introduced, which revealed the presence of dynamic characteristics in the system. It was found that the Lyapunov values were: LE_1 = 0.315207, LE_2 = 0.135137 and LE_3 = -0.082011, LE_4 =-0.275085, and LE_5 =-11.9927.

Another hyper-chaotic 5D system was developed in [13] to ensure communication. Important features, such as stability at equilibrium and LEs, were carefully analyzed and the results were: Ly_1 = 0.17826, Ly_2 = 0.110013, Ly_3 = 0, Ly_4 = − 5.18539, and Ly_5 = − 28.108.

Authors in [14] presented a 5D hyper-chaotic system that exhibits intriguing and intricate behaviors. The Lyapunov method was employed to demonstrate the stability of the controller in both scenarios. Authors in [15] proposed a new continuous chaotic system centered on Field Programmable Gate Arrays (FPGAs). The system comprised 5 dimensions, with dynamics enabling the use of chaotic signals in cryptography applications. The LEs of the system were: Ly_1 = 0.093790, Ly_2 = 0.001101, Ly_3 = 0.000107, Ly_4 = –0.500100, and Ly_5 = –0.594898. Authors in [16] introduced a model for a unique 5D hyper-chaotic system featuring three positive LEs. The largest positive LE exceeds 12: Ly_1 = 12.659, Ly_2 = 0.055, Ly_3 = 0.024, Ly_4 = 0, and Ly_5 = –67.7001.

It becomes clear that most of the existing 5D systems in the available related work either have two LEs or a small value of LEs. Therefore, there is a need to develop a more convenient and valid 5D hyper-chaotic cipher system. For this purpose, this paper proposes a novel 5D hyper-chaotic cipher system with different multilayer chaotic attractors. The attributes of the new system's dynamic behavior are investigated using the proposed mathematical program.

II. ESTABLISHMENT OF THE NEW FIVE-DIMENSIONAL DYNAMIC SYSTEM

A hyper-chaotic system can be characterized as a chaotic system which has at least 2 positive LEs [17]. This means that the system's chaotic dynamics are expanded in multiple directions, creating more complex attractors.

The main goal is to design a mathematical model that represents a novel chaotic cipher system. The new 5D autonomous system is represented by:

$$\begin{aligned}
\frac{dx_1}{dt} &= -a\cdot x_1 + b\cdot x_2 + c\cdot x_3x_4 - x_3x_5 \\
\frac{dx_2}{dt} &= -x_2 + d\cdot x_1 - e\cdot x_1x_3 - f\cdot x_5 \\
\frac{dx_3}{dt} &= -g\cdot x_3 + f\cdot x_1x_2 + x_4 \\
\frac{dx_4}{dt} &= -h\cdot x_4 - i\cdot x_1x_3 + j\cdot x_5 \\
\frac{dx_5}{dt} &= -b\cdot x_5 + a\cdot x_2x_3 - x_1x_3
\end{aligned} \tag{1}$$

where x_1, x_2, x_3, x_4, x_5, and t are real numbers representing system states, while a, b, c, d, e, f, g, h, i, and j are the system's positive parameters.

In the case where system parameter values and initial conditions are chosen as follows, the newly created 5D system (1) has displayed a chaotic attractor:

$$a\in[4,\ 5],\ b\in[2,\ 3],\ c\in[2.5,\ 3.5],\ d\in[19,\ 20],$$
$$e\in[1.2,\ 2.2],\ f\in[0,\ 0.5],\ g\in[3.5,\ 4.5],$$
$$h\in[0.5,\ 2],\ i\in[1.3,\ 2.3],\ j\in[0,\ 1.2].$$

$$x_1(0)=0.1,\ x_2(0)=0.3,\ x_3(0)=0.4,$$
$$x_4(0)=0,\ x_5(0)=0.01.$$

This system with its characteristics displays a variety of intricate and unpredictable chaotic behaviors. Its structure, as depicted in Figures 1, 2, resembles the front part of a ship, leading to the introduction of the term "Butterfly Effect."

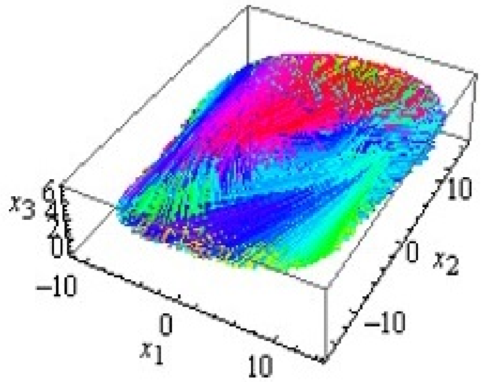

Fig. 1. Chaotic attractors, three-dimensional view (x_1, x_2, x_3).

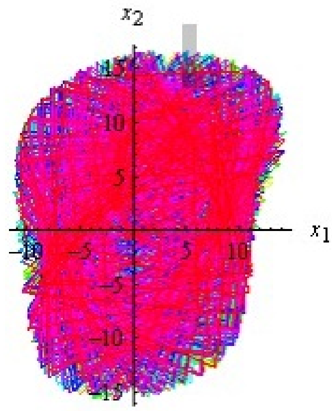

Fig. 2. Chaotic attractors, (x_1, x_2) phase plane.

III. DYNAMIC ANALYSIS OF A NEW HYPER-CHAOTIC SYSTEM

A. *Dissipativity*

The new system (1) is represented by:

$$F=\begin{bmatrix} f_1\\ f_2\\ f_3\\ f_4\\ f_5 \end{bmatrix} = \begin{bmatrix} \frac{dx_1}{dt}\\ \frac{dx_2}{dt}\\ \frac{dx_3}{dt}\\ \frac{dx_4}{dt}\\ \frac{dx_5}{dt} \end{bmatrix} = \begin{bmatrix} -a\cdot x_1 + b\cdot x_2 + c\cdot x_3x_4 - x_3x_5\\ -x_2 + d\cdot x_1 - e\cdot x_1x_3 - f\cdot x_5\\ -g\cdot x_3 + f\cdot x_1x_2 + x_4\\ -h\cdot x_4 - i\cdot x_1x_3 + j\cdot x_5\\ -b\cdot x_5 + a\cdot x_2x_3 - x_1x_3 \end{bmatrix} \tag{2}$$

The vector field divergence f on R^5 is represented by:

$$\nabla\cdot F=\frac{\partial f_1}{\partial x_1}+\frac{\partial f_2}{\partial x_2}+\frac{\partial f_3}{\partial x_3}+\frac{\partial f_4}{\partial x_4}+\frac{\partial f_5}{\partial x_5} \quad (3)$$

Equation (3) measures the rate at which the volume values change under the flow Φ_t of F. Assuming D is a region in R^5 with a smooth boundary for which it applies that $D(t)=\Phi_t(t)$, then D(t) is the image of D under Φ_t in time t of the F flow. The volume V(t) of D(t), according to the Liouville's theorem [18], is calculated by:

$$\frac{dV}{dt}=\int_{D(t)}\left(\nabla\cdot F\right)dx_1dx_2dx_3dx_4dx_5 \quad (4)$$

For system (1), from (3), it is calculated that:

$$\nabla\cdot F=-\left(a+1+g+h+b\right)<0 \quad (5)$$

Because a, b, g, and h are positive constants, (4) with the use of (5) gives:

$$\begin{aligned}&\frac{dV}{dt}=-\left(a+1+g+h+b\right)\\&\int_{D(t)}dx_1dx_2dx_3dx_4dx_5 \Leftrightarrow\\&\frac{dV}{dt}=-\left(a+1+g+h+b\right)V\left(t\right)\Leftrightarrow\\&\frac{dV}{dt}=e^{-14.3t}V\left(t\right)\end{aligned} \quad (6)$$

Solving the 1st order linear differential equation (6), the result is the unique solution:

$$V\left(t\right)=V\left(0\right)e^{-(a+1+g+h+b)t}=V\left(0\right)e^{-14.3t} \quad (7)$$

Any volume V(t) should decrease in an exponential manner quickly to zero over time, as demonstrated by (7). As a result, the dynamical system that (1) describes is a dissipative system.

B. Symmetry and Invariability

If coordinates (x_1, x_2, x_3, x_4, x_5) are transformed to ($-x_1$, $-x_2$, x_3, $-x_4$, $-x_5$), the new system is invariant and has a symmetry around the x_3 and p-axes.

Assuming that:

$$x_1=-x_1,\ x_2=-x_2,\ x_3=x_3,\ x_4=-x_4,\ x_5=-x_5 \quad (8)$$

It gives:

$$\begin{aligned}&\frac{dx_1}{dt}=-\frac{dx_1}{dt},\ \frac{dx_2}{dt}=-\frac{dx_2}{dt},\ \frac{dx_3}{dt}=\frac{dx_3}{dt},\\&\frac{dx_4}{dt}=-\frac{dx_4}{dt},\ \frac{dx_5}{dt}=-\frac{dx_5}{dt}\end{aligned} \quad (9)$$

Using (8) and (9) in (1) the result is:

$$\begin{aligned}&-\frac{dx_1}{dt}=a\cdot x_1-b\cdot x_2-c\cdot x_3x_4+x_3x_5\\&-\frac{dx_2}{dt}=x_2-d\cdot x_1+e\cdot x_1x_3+f\cdot x_5\\&\frac{dx_3}{dt}=-g\cdot x_3+f\cdot x_1x_2+x_4\\&-\frac{dx_4}{dt}=h\cdot x_4+i\cdot x_1x_3-j\cdot x_5\\&-\frac{dx_5}{dt}=b\cdot x_5-a\cdot x_2x_3+x_1x_3\end{aligned} \quad (10)$$

Equation (10) ends to (11):

$$\begin{aligned}&\frac{dx_1}{dt}=-a\cdot x_1+b\cdot x_2+c\cdot x_3x_4-x_3x_5\\&\frac{dx_2}{dt}=-x_2+d\cdot x_1-e\cdot x_1x_3-f\cdot x_5\\&\frac{dx_3}{dt}=-g\cdot x_3+f\cdot x_1x_2+x_4\\&\frac{dx_4}{dt}=-h\cdot x_4-i\cdot x_1x_3+j\cdot x_5\\&\frac{dx_5}{dt}=-b\cdot x_5+a\cdot x_2x_3-x_1x_3\end{aligned} \quad (11)$$

Equation (11) is the same as (1), so the system remains unchanged to the coordinate's transformation. Therefore, the system in (1) has a symmetry of rotation about the x_3-axis. In addition, any nontrivial trajectory of the system should have a twin trajectory. Also, the x_3-axis is invariant under the system's flow.

C. Equilibrium Points

The study searches for 2 points of equilibrium when:

a=5, b=3, c=3.5, d=20, e=2.2, f=0.5, g=4.5, h=0.8, i=2.3, j=1.2.

Based on system (1) results, the nonlinear equations are:

$$\begin{aligned}&0=-a\cdot x_1+b\cdot x_2+c\cdot x_3x_4-x_3x_5\\&0=-x_2+d\cdot x_1-e\cdot x_1x_3-f\cdot x_5\\&0=-g\cdot x_3+f\cdot x_1x_2+x_4\\&0=-h\cdot x_4-i\cdot x_1x_3+j\cdot x_5\\&0=-b\cdot x_5+a\cdot x_2x_3-x_1x_3\end{aligned} \quad (12)$$

Solving (12) gives the two potential equilibrium points: E_0(0, 0, 0, 0, 0) and E_1(0, 0, 4.98814, 0, 0). The Jacobian matrix of system (1) is calculated in:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \frac{\partial f_2}{\partial x_5} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} & \frac{\partial f_3}{\partial x_5} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} & \frac{\partial f_4}{\partial x_5} \\ \frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_3} & \frac{\partial f_5}{\partial x_4} & \frac{\partial f_5}{\partial x_5} \end{bmatrix} \Leftrightarrow$$

$$J = \begin{bmatrix} -a & b & cx_4 - x_5 & cx_3 & x_3 \\ d - ex_3 & -1 & -ex_1 & 0 & -f \\ fx_2 & fx_1 & -g & 1 & 0 \\ -ix_3 & 0 & -ix_1 & -h & j \\ -x_3 & ax_3 & ax_2 - x_1 & 0 & -b \end{bmatrix} \quad (13)$$

For the equilibrium point $E_0(0, 0, 0, 0, 0)$, the Jacobian matrix gives:

$$J_0 = \begin{bmatrix} -5 & 3 & 0 & 0 & 0 \\ 20 & -1 & 0 & 0 & -0.5 \\ 0 & 0 & -4.5 & 1 & 0 \\ 0 & 0 & 0 & -0.8 & 1.2 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix} \quad (14)$$

The next step is to find the eigenvalues by solving:

$$|\lambda I - J_0| = 0 \quad (15)$$

The results are:

$$\lambda_1 = -11, \lambda_2 = 5, \lambda_3 = -4.5, \lambda_4 = -3, \lambda_5 = -0.8$$

The new system starting from point E_0 includes both negative and positive real numbers, making the E_1 equilibrium point unstable. The Jacobian matrix for E_1 gives:

$$J_1 = \begin{bmatrix} -5 & 3 & 0 & 3.5\cdot 4.9881 & -4.9881 \\ 20\cdot 4.9881 & -1 & 0 & 0 & -0.5 \\ 0 & 0 & -4.5 & 1 & 0 \\ -2.3\cdot 4.9881 & 0 & 0 & -0.8 & 1.2 \\ -4.9881 & 5\cdot 4.9881 & 0 & 0 & -3 \end{bmatrix}$$

The eigenvalues for E_1 are:

$$\lambda_1 = -28.9002, \lambda_2 = 7.91152 + 18.8782i,$$
$$\lambda_3 = 7.91152 - 18.8772i, \lambda_4 = -4.5, \lambda_5 = 3.27711.$$

where λ_2 and λ_3 are complex numbers, so E_1 is a saddle point. Therefore, such a point of equilibrium is not stable.

D. Lyapunov Exponents and Lyapunov Dimensions

The LE is calculated using the quantitative measure technique of sensitive dependency on the initial conditions, according the nonlinear dynamical theory. It represents the average rate at which two nearby trajectories are diverging (or converging). Using the same values for the parameters a, b, c, d, e, f, g, h, i, and j, the LEs for the system (1) are: LE_1=4.03832, LE_2=3.5148, LE_3=6.85408, LE_4=-5.89531, and LE_5=-22.8248.

The MLE (LE_3) is positive, so the system exhibits chaotic properties. Additionally, the system is hyper-chaotic according to:

$$LE_1 + LE_2 + LE_3 + LE_4 + LE_5 < 0 \quad (16)$$

$$LE_1 + LE_2 + LE_3 + LE_4 > 0 \quad (17)$$

Another common feature of chaos is the fractal dimension, which is the Kaplan-Yorke dimension (D_{KY}) determined through the LEs. D_{KY} is calculated by:

$$D_{KY} = j + \frac{1}{|LE_{j+1}|} \sum_{i=1}^{j} LE_i \quad (18)$$

where j is the largest index for which it applies:

$$\sum_{i=1}^{j} LE_i \geq 0 \text{ and } \sum_{i=1}^{j+1} LE_i < 0.$$

Therefore, according to (16), (17) the j value is 4, so (18) results in/the result is:

$$D_{KY} = 4 + \frac{1}{|LE_5|} \sum_{i=1}^{4} LE_i = 4 + \frac{LE_1 + LE_2 + LE_3 + LE_4}{|LE_5|} =$$
$$4 + \frac{4.03832 + 3.5148 + 6.85408 + (-5.89531)}{22.8248} = 4.37292$$

This indicates that the Lyapunov dimension for system (1) is fractional. The new system includes the non-periodic orbits and nearby trajectories diverging as a result of its fractal character, which is why this non-linear system is a real chaos.

IV. PHASE PORTRAITS

The dynamical behavior of the novel hyper-chaotic attractor appears to be exceedingly complex, chaotic, and intriguing. Figures 3and 4 present the phase portraits.

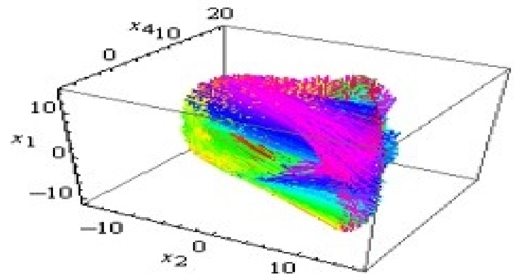

Fig. 3. Chaotic attractors, three-dimensional view (x_1, x_2, x_4).

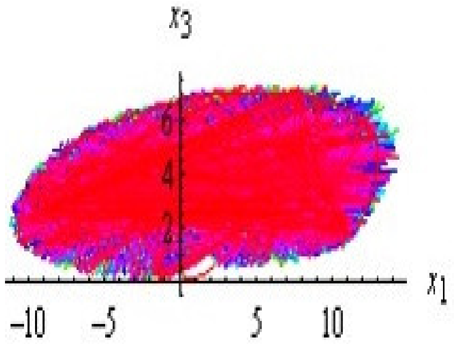

Fig. 4. Chaotic attractors, two-dimensional view (x_3, x_1) phase plane.

V. WAVEFORM ANALYSES OF THE NOVEL CHAOTIC SYSTEM

A chaotic system waveform must be a periodic. To demonstrate that the proposed system is a hyper-chaotic system, wave-forms of $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, and $x_5(t)$ have been exhibited in the time domain, as can be seen in Figure 5. For the purpose of discriminating between multiple periodic motions that can also exhibit chaotic motions and a complicated behavior, it could be noted that the time domain waveform has noncyclical characteristics.

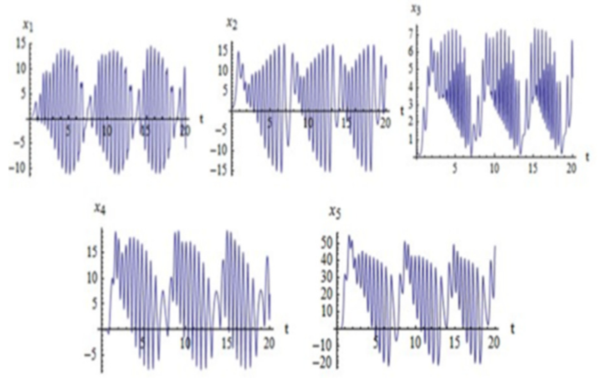

Fig. 5. Time versus x_1, x_2, x_3, x_4, x_5 of the novel chaotic system.

VI. INITIAL CONDITIONS SENSITIVITY

The defining aspect of a chaotic system is its long-term unpredictability, stemming from the sensitivity of the solutions to the initial conditions. Even slight differences in the initial conditions will lead to significant divergence over time [19]. As a result, there will be a point in the future where precise predictions about the system states will become impossible with any level of accuracy in the initial conditions.

Two cases were examined in order to present that the chaos trajectory evolution has extreme sensitivity to the initial conditions. Figure 6 illustrates the blue solid line with initial conditions: $x_1(0) = 0.1$, $x_2(0) = 0.3$, $x_3(0) = 0.4$, $x_4(0) = 0$, and $x_5(0) = 0.01$, and the red dashed line with initial conditions: $x_1(0) = 0.1$, $x_2(0) = 0.3000000000000001$, $x_3(0) = 0.4$, $x_4(0) = 0$, and $x_5(0) = 0.01$.

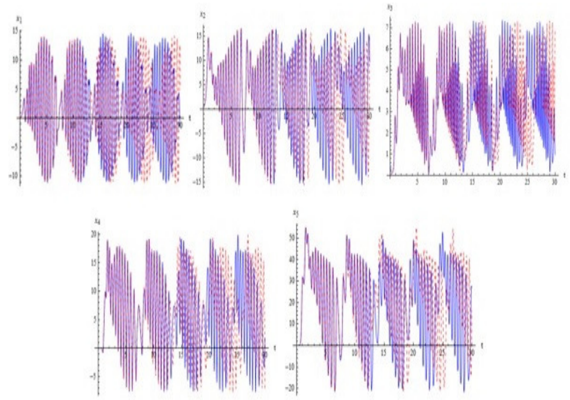

Fig. 6. Sensitivity tests of the novel system $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, $x_5(t)$.

VII. BIFURCATION DIAGRAM

The new system (1) is numerically solved through the simulation of the proposed mathematical program. By taking the maximum values of the variable x_2 and changing the values of the parameter a, the interesting region of x_2 can be clearly observed in which the system changes its behavior and the values of x_2 begin to bifurcate, especially between 5.15 and 5.20. Additionally, at the values between 5.30 and 5.35, the bifurcation property seems to be a significant characteristic of the chaotic systems, as displayed in Figure 7.

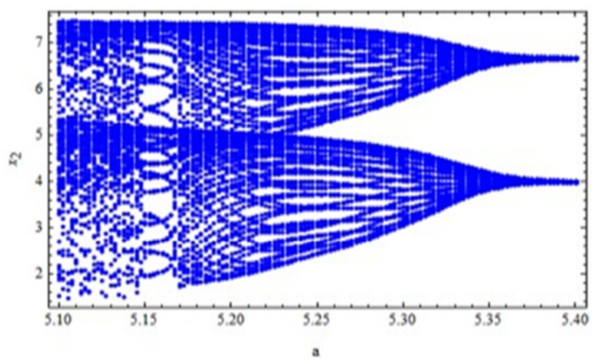

Fig. 7. Bifurcation diagram of the variable x_2 in relation to the parameter a.

VIII. COMPARISON OF CHAOTIC SYSTEMS

Chaotic systems have a vital role in many applications, especially in the fields of secure communications, cryptography, and the mathematical representation of complex natural phenomena. In comparison to traditional systems, the proposed 5D chaotic system is a sophisticated model that offers a larger degree of unpredictability and complexity. Hence it has the potential to enhance security applications. Table I provides a comprehensive overview of the distinctive features of the proposed system in comparison to the Lorenz, Rössler, and Chua systems [20-23] and highlights its chaotic nature and various potential applications .

TABLE I. COMPARATIVE ANALYSIS OF THE 5D PROPOSED SYSTEM WITH OTHER CHAOTIC SYSTEMS

Chaotic systems	**Dimensions**	**Level of complexity**	**Chaotic behaviors**	**Potential applications**
5D proposed system	Five dimensions	Complex attractors	A high level of chaos associated with an increase in unpredictability	Encryption, communications, security
Lorenz System	Three dimensions	A number of attractors that have less complicated shapes	The level of chaos is modest and highly responsive to the initial conditions	Accurate climatic forecasts, fluid motion
Rössler Attractor	Three dimensions	A moderate level of complexity	Chaotic dynamics similar to those in Lorenz but with a smoother behavior	Chemical processes, modeling of ecosystems
Chua's Circuit	Three dimensions	A moderate level of complexity	Identical chaotic dynamics to those observed in Lorenz, but exhibiting smoother behavior	Design of electronic circuits, establishment of dependable communication

IX. CONCLUSION

This study proposed a novel Five-Dimensional (5D) hyper-chaotic system, which has 5 Lyapunov Exponents (LEs) (LE_1=4.03832, LE_2=3.5148, LE_3=6.85408, LE_4=-5.89531, and LE_5=-22.8248), complex dynamical behaviors, and all of the chaotic system's characteristics. It appears to be more convenient and valid in comparison to the existing 5D systems in the reviewed related work as most of them either have two LEs, or a small value of LEs. Also, the current system has a higher value of LEs compared to the available aforementioned systems. The investigation of the fundamental properties and dynamic behavior of the new system has revealed its chaotic nature. This is evidenced by the presence of 2 points of unstable equilibrium, the existence of 2 positive LEs, and a fractal dimension of 4.37292. Additionally, the system exhibits strong sensitivity characteristics, generates a complex chaotic attractor, has a bifurcation property, and has a very large key space. These collective findings strongly suggest that the system can be classified as hyper-chaotic. In future work, it is recommended that this 5D approach be used to generate random keys for data encryption.

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