Comparing Subjective Weighting Methods in Multi-Criteria Decision-Making: An Application to Electric Bicycle Ranking

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ABSTRACT

Ranking the various electric bicycle models available in the market, each with different specifications, is a complex task. The importance of criteria in this process depends on subjective weighting methodsbecause the assigned weights to the criteria are based on the decision-maker's subjective priorities. This study compares three subjective weighting methods, namely the Rank Order Centroid (ROC) method, the Rank Sum (RS) method, and a method based on the Lagrange multiplier (referred to as the Lagrange method). These methods share the common characteristic of deriving weights from the evaluation of criteria, yet they differ in their specific formulas. The three methods were applied to assign weights to the criteria used in evaluating seven electric bicycle models across 10 different criteria. The weights were calculated under 10 different scenarios, each reflecting a change in the prioritization of criteria. For each scenario, four Multi-Criteria Decision-Making (MCDM) methods were used to rank the electric bicycles: Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Ranking of Alternatives with Weights of Criterion (RAWEC), Particle Image Velocimetry (PIV), and Root Assessment Method (RAM). The comparison of weighting methods was based on the average Spearman rank correlation coefficient between the MCDM rankings obtained using different weighting methods. The findings indicate that the ROC and Lagrange methods outperformed the RS method.

Keywords-MCDM; subjective weighting; electric bicycle ranking; Spearman rank correlation coefficient

I. INTRODUCTION

In modern cities, electric bikes (e-bikes) are getting very popular for being convenient, friendly to the environment and low-cost. There are many e-bikes, with different characteristics and specifications, making the selection of the most suitable very difficult. A ranking system would be helpful in simplifying this process. However, the variety of factors involved, including ergonomics, battery capacity, and cost, complicates the task of determining the best choice. MCDM has become a popular solution for such challenges [1-3]. Weighting the criteria of an e-bike is an important step in making a smart purchase decision [4-6]. Having this in mind, consumers often use qualitative methods as a criterion resulting in a unique set of weights for each consumer, reflecting their individual needs and preferences. For instance, one buyer may prefer travel distance over maximum speed, while another may value more the braking system than the design. By assigning weights to the criteria, consumers can easily compare different e-bikes and choose the one that best suits their needs. Moreover, the use of weighting methods helps them focus on the factors that are truly important, instead of being distracted by unnecessary technical specifications. This makes the e-bike selection process simpler and more efficient. However, the direct assignment of weights to the criteria based on subjective opinions does not guarantee accuracy. Typically, decision-makers feel more confident when determining the ranks of the criteria as opposed to specifying their weights [7]. Moreover, the methods of calculating weights based on criteria ranking are proven to be far superior to directly assigning weights to the criteria when the decision-makers do not reach a consensus on assigning weights, but they can only reach a consensus on the relative importance of the criteria [8, 9]. In some cases, the calculation of weights for criteria based on the priority rank, rather than the direct assignment of weights, can also select more feasible options from among many options that need to be ranked. This is considered a prominent advantage of calculating subjective weights based on the priority rank of the criteria instead of the direct assignment of weights to the criteria [10]. ROC, RS, and Lagrange are three of the main subjective weighting methods, with ROC and RS being the most used ones [11, 12]. The ROC method is frequently used to calculate weights for criteria based on specifying the importance of ranking metal cutting solutions [13], selecting sellers on e-marketplaces [14], choosing water loss prevention solutions in Tanzania [15], choosing engines [16], controlling automatic power generation systems [17] and selecting subcontractors in the construction of nuclear power plants [18]. The combination of ROC and RS is used in the ranking of metal milling solutions and the ranking of office air quality [19], metal cutting solutions [20, 21], the selection of transportation options [22] and the selection of the best solution for milling, grinding, and turning metal [23]. In contrast, the application of the Lagrange weighting method remains very limited, used only in the selection of wheel loaders [24, 25]. The existing literature creates a gap in the effectiveness of the three methods, while this study aims to fill this gap by examining the relative advantages of the methods. The scope of the present study is limited to the evaluation of electric bicycle models, a topic of contemporary importance.

II. MATERIALS AND METHODS

A. Electric Bicycle Models to be Ranked

Table I shows the data for the seven electric bicycle models that will be ranked from A1 to A7 [26]. Ten criteria were used to describe each model, price (C1), the distance the bike could travel on a single charge (C2), charging time (C3), maximum speed (C4), the bikes' weight (C5), the bikes' load capacity (C6), seat height (C7) the overall length of the bike (C8), the overall width of the bike (C9) and the overall height of the bike (C10). Two of these, C1 and C3 are the C-type criteria, while all the other are the B-type criteria. It is clear that we cannot rank the options based only on the data in Table I, because no option guarantees that both C1 and C3 will have the smallest values, and that all eight other criteria will have the largest values. In order to rank the alternatives, we need to calculate the weights for the criteria and use MCDM methods.

B. Subjective Weighting Methods Used

The ROC and the RS method are used to calculate the weights for the criteria:

$$w_j = \frac{1}{n}\sum_k^n \frac{1}{k} \quad (1)$$

$$w_j = \frac{2(n+1-k)}{n(n+1)} \quad (2)$$

where k is the order of priority of criterion j, n is the number of criteria, and j = 1 to n [19-21].

TABLE I. ELECTRIC BICYCLE MODELS

Alt.	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	min	max	min	max	max	max	max	max	max	max
A1	7500000	45	7	35	56	160	750	1593	635	1015
A2	7900000	45	7	35	50	150	750	1640	640	1200
A3	9900000	50	7	35	50	180	750	1640	640	1200
A4	9900000	50	7	35	46	180	750	1640	640	1200
A5	11500000	50	7	35	52	180	750	1640	640	1200
A6	13990000	45	7	30	45	75	550	1550	650	1040
A7	13990000	45	8	30	45	75	600	1530	750	1000

The steps for calculating the weights of the criteria using the Lagrange method are [24, 25]:

- Decide on the order of priority for the criteria, with the most important ones at the top.
- Calculate the Lagrange multiplier using:

$$\lambda = \frac{2}{\left(\sum_{j=1}^{n}\frac{1}{[(n+1)-j]}\right)[n(n+1)]} \quad (3)$$

- Calculate the weights of the criteria using:

$$w_j = \frac{\lambda[n(n+1)]}{2[(n+1)-j]} \quad (4)$$

C. MCDM Methods

In this study, four MCDM methods were used: TOPSIS, RAWEC, PIV, and RAM. TOPSIS was selected because it is the most used method, generally [27], RAWEC because it is one of the few methods that simultaneously uses two types of data normalization [28], PIV due to its ability to minimize the reversal of ranks [29, 30] and RAM because it can balance between benefit and cost criteria [31]. The TOPSIS method is used with the following steps:

- Determine the normalized values:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}} \quad (5)$$

- Calculate the weighted normalized values:

$$Y = w_j . n_{ij} \quad (6)$$

where m is the number of alternatives to be ranked, n is the number of criteria for each alternative, x_{ij} is the value of criterion j at alternative i, with j = 1 to n, and i = 1 to m.

- Determine the best solution A+ and the worst solution A- for the criteria:

$$A^+ = \{y_1^+, y_2^+, \dots, y_j^+, \dots, y_n^+\} \quad (7)$$

$$A^- = \{y_1^-, y_2^-, \dots, y_j^-, \dots, y_n^-\} \quad (8)$$

where y_j^+ and y_j^- are respectively the best value and the worst value of the weighted normalized value of criterion j.

- Determine the values S_i^+ and S_i^-:

$$S_i^+ = \sqrt{\sum_{j=1}^{n}\left(y_{ij} - y_j^+\right)^2} \quad (9)$$

$$S_i^- = \sqrt{\sum_{j=1}^{n}\left(y_{ij} - y_j^-\right)^2} \quad (10)$$

- Determine the scores of the alternatives, ranking the alternatives according to the principle that the alternative with the highest score is the best alternative:

$$C_i^* = \frac{S_i^-}{S_i^+ + S_i^-} \quad (11)$$

The RAWEC method is ranking the alternatives using the following [28]:

- Double normalization according to:

$$n_{ij} = \frac{x_{ij}}{max(x_{ij})}, \text{ and } n'_{ij} = \frac{\min(x_{ij})}{x_{ij}}, \text{ if } j \in B \quad (12)$$

$$n_{ij} = \frac{\min(x_{ij})}{x_{ij}}, \text{ and } n'_{ij} = \frac{x_{ij}}{max(x_{ij})}, \text{ if } j \in C \quad (13)$$

- The deviation from the weights of the criteria is:

$$v_{ij} = \sum_{j=1}^{n} w_j \cdot (1 - n_{ij}) \quad (14)$$

$$v'_{ij} = \sum_{j=1}^{n} w_j \cdot (1 - n'_{ij}) \quad (15)$$

- Calculate the scores of the alternatives ranking them in the way that 1 is the alternative with the highest score:

$$Q_i = \frac{v'_{ij} - v_{ij}}{v'_{ij} + v_{ij}} \quad (16)$$

The PIV method of ranking follows the procedure:

- Calculate the normalized values according to (5).
- Calculate the weighted normalized values of the criteria according to (6).
- Evaluate the weighted proximity index according to:

$$u_i = Y_{\max} - Y_i, \text{ if } j \in B \quad (17)$$

$$u_i = Y_i - Y_{\min}, \text{ if } j \in C \quad (18)$$

- Determine the scores of the alternatives ranking them to the principle that the best alternative is the alternative with the smallest score:

$$d_i = \sum_{j=1}^{n} u_i \quad (19)$$

The RAM method of ranking uses the following steps [31]:

- Normalize the data according to:

$$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad (20)$$

- Calculate the weighted normalized values of the criteria:

$$y_{ij} = w_j \cdot n_{ij} \quad (21)$$

- Calculate the total weighted normalized score of the criteria:

$$S_{+i} = \sum_{j=1}^{n} y_{+ij}, \text{ if } j \in B \quad (22)$$

$$S_{-i} = \sum_{j=1}^{n} y_{-ij}, \text{ if } j \in C \quad (23)$$

- Calculate the score of each alternative, ranking them in descending order of their scores:

$$RI_i = \sqrt[2+S_{-i}]{2 + S_{+i}} \quad (24)$$

III. RESULTS AND DISCUSSION

In order to compare the three subjective weighting methods -ROC, RS, and Lagrange - the ten criteria of the e-bikes were measured under different scenarios. Accordingly, each criterion was selected to have a priority ranking of 1 once, while the remaining criteria had corresponding priority levels in their order. For scenario S1, criterion C1 is prioritized as 1, criterion C2 as 2, criterion C3 as 3, and so on, with criterion C10 receiving the highest priority. For scenario S2, criterion C2 has the highest priority, followed by C1, C3, and so on, with criterion C10 ranked as 10. The details of these ten scenarios are shown in Table II.

TABLE II. DIFFERENT SCENARIOS

Scenarios	Criteria ranking in descending order of priority
S1	C1 > C2 > C3 > C4 > C5 >C6 > C7 > C8 > C9 > C10
S2	C2 > C1 > C3 > C4 > C5 >C6 > C7 > C8 > C9 > C10
S3	C3 > C1 > C2 > C4 > C5 >C6 > C7 > C8 > C9 > C10
S4	C4 > C1 > C2 > C3 > C5 >C6 > C7 > C8 > C9 > C10
S5	C5 > C1 > C2 > C3 > C4 >C6 > C7 > C8 > C9 > C10
S6	C6 > C1 > C2 > C3 > C4 >C5 > C7 > C8 > C9 > C10
S7	C7 > C1 > C2 > C3 > C4 >C5 > C6 > C8 > C9 > C10
S8	C8 > C1 > C2 > C3 > C4 >C5 > C6 > C7 > C9 > C10
S9	C9 > C1 > C2 > C3 > C4 >C5 > C6 > C7 > C8 > C10
S10	C10 > C1 > C2 > C3 > C4 >C5 > C6 > C7 > C8 > C9

The weights of the criteria for all 10 scenarios were calculated by applying the ROC, RS, and Lagrange methods as shown in Table III. The TOPSIS, RAWEC, PIV, and RAM methods were then used to rank the e-bike models in each scenario. Table IV presents the scores and ranks of the alternatives when using the TOPSIS, RAWEC, PIV, and RAM methods in scenario S1. In this, S1 uses the ROC or Lagrange method to calculate the weights for the criteria, and the TOPSIS, RAWEC, PIV, or RAM method is applied to rank the e-bikes. This results in A1 receiving the highest rank, A2 the second, A3 the third-highest and so on till A7. If the RS method is used, the rank of the alternatives remains consistent when using the three methods TOPSIS, RAWEC, and PIV. However, when the RAM method is used the rank of the alternatives differs when compared to the rank based on TOPSIS, RAWEC, and PIV. Regardless of the method used in ranking, A1 is identified as the optimal alternative, while A5 ranks fifth, A6 sixth, and A7 seventh. The Spearman rank correlation coefficient was used to compare the ROC, RS, and Lagrange method:

$$S = 1 - \frac{6\sum_{i=1}^{m} D_i^2}{m(m^2-1)} \quad (25)$$

where *Di* is the difference in rank of alternative *i* by different methods [32, 33].

TABLE III. WEIGHTS OF CRITERIA

Scenario	Weight method	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
S1	ROC	0.2929	0.1929	0.1429	0.1096	0.0846	0.0646	0.0479	0.0336	0.0211	0.0100
	RS	0.1818	0.1636	0.1455	0.1273	0.1091	0.0909	0.0727	0.0545	0.0364	0.0182
	Lagrange	0.3414	0.1707	0.1138	0.0854	0.0683	0.0569	0.0488	0.0427	0.0379	0.0341
S2	ROC	0.1929	0.2929	0.1429	0.1096	0.0846	0.0646	0.0479	0.0336	0.0211	0.0100
	RS	0.1636	0.1818	0.1455	0.1273	0.1091	0.0909	0.0727	0.0545	0.0364	0.0182
	Lagrange	0.1707	0.3414	0.1138	0.0854	0.0683	0.0569	0.0488	0.0427	0.0379	0.0341
S3	ROC	0.1929	0.1429	0.2929	0.1096	0.0846	0.0646	0.0479	0.0336	0.0211	0.0100
	RS	0.1636	0.1455	0.1818	0.1273	0.1091	0.0909	0.0727	0.0545	0.0364	0.0182
	Lagrange	0.1707	0.1138	0.3414	0.0854	0.0683	0.0569	0.0488	0.0427	0.0379	0.0341
S4	ROC	0.1929	0.1429	0.1096	0.2929	0.0846	0.0646	0.0479	0.0336	0.0211	0.0100
	RS	0.1636	0.1455	0.1273	0.1818	0.1091	0.0909	0.0727	0.0545	0.0364	0.0182
	Lagrange	0.1707	0.1138	0.0854	0.3414	0.0683	0.0569	0.0488	0.0427	0.0379	0.0341
S5	ROC	0.1929	0.1429	0.1096	0.0846	0.2929	0.0646	0.0479	0.0336	0.0211	0.0100
	RS	0.1636	0.1455	0.1273	0.1091	0.1818	0.0909	0.0727	0.0545	0.0364	0.0182
	Lagrange	0.1707	0.1138	0.0854	0.0683	0.3414	0.0569	0.0488	0.0427	0.0379	0.0341
S6	ROC	0.1929	0.1429	0.1096	0.0846	0.0646	0.2929	0.0479	0.0336	0.0211	0.0100
	RS	0.1636	0.1455	0.1273	0.1091	0.0909	0.1818	0.0727	0.0545	0.0364	0.0182
	Lagrange	0.1707	0.1138	0.0854	0.0683	0.0569	0.3414	0.0488	0.0427	0.0379	0.0341
S7	ROC	0.1929	0.1429	0.1096	0.0846	0.0646	0.0479	0.2929	0.0336	0.0211	0.0100
	RS	0.1636	0.1455	0.1273	0.1091	0.0909	0.0727	0.1818	0.0545	0.0364	0.0182
	Lagrange	0.1707	0.1138	0.0854	0.0683	0.0569	0.0488	0.3414	0.0427	0.0379	0.0341
S8	ROC	0.1929	0.1429	0.1096	0.0846	0.0646	0.0479	0.0336	0.2929	0.0211	0.0100
	RS	0.1636	0.1455	0.1273	0.1091	0.0909	0.0727	0.0545	0.1818	0.0364	0.0182
	Lagrange	0.1707	0.1138	0.0854	0.0683	0.0569	0.0488	0.0427	0.3414	0.0379	0.0341
S9	ROC	0.1929	0.1429	0.1096	0.0846	0.0646	0.0479	0.0336	0.0211	0.2929	0.0100
	RS	0.1636	0.1455	0.1273	0.1091	0.0909	0.0727	0.0545	0.0364	0.1818	0.0182
	Lagrange	0.1707	0.1138	0.0854	0.0683	0.0569	0.0488	0.0427	0.0379	0.3414	0.0341
S10	ROC	0.1929	0.1429	0.1096	0.0846	0.0646	0.0479	0.0336	0.0211	0.0100	0.2929
	RS	0.1636	0.1455	0.1273	0.1091	0.0909	0.0727	0.0545	0.0364	0.0182	0.1818
	Lagrange	0.1707	0.1138	0.0854	0.0683	0.0569	0.0488	0.0427	0.0379	0.0341	0.3414

TABLE IV. RANKING OF ALTERNATIVES IN SCENARIO S1

Alt.	TOPSIS		RAWEC		PIV		RAM	
	*Ci**	Rank	*Qi*	Rank	*di*	Rank	*RIi*	Rank
ROC Method								
A1	0.8891	1	0.7585	1	0.0134	1	1.4306	1
A2	0.8552	2	0.5862	2	0.0219	2	1.4296	2
A3	0.6556	3	0.4155	3	0.0295	3	1.4293	3
A4	0.6502	4	0.3718	4	0.0321	4	1.4289	4
A5	0.4532	5	0.2123	5	0.0444	5	1.4279	5
A6	0.0985	6	-0.8543	6	0.1114	6	1.4202	6
A7	0.0266	7	-0.9452	7	0.1167	7	1.4197	7
RS Method								
A1	0.8501	1	0.7072	1	0.0153	1	1.4389	1
A2	0.7966	2	0.5489	2	0.0233	3	1.4380	4
A3	0.7054	3	0.5251	3	0.0224	2	1.4385	2
A4	0.6872	4	0.4629	4	0.0257	4	1.4380	3
A5	0.5634	5	0.4023	5	0.0308	5	1.4377	5
A6	0.1337	6	-0.8322	6	0.0998	6	1.4293	6
A7	0.0581	7	-0.9048	7	0.1040	7	1.4290	7
Lagrange Method								
A1	0.9060	1	0.7444	1	0.0148	1	1.4291	1
A2	0.8783	2	0.6073	2	0.0214	2	1.4284	2
A3	0.6452	3	0.3641	3	0.0338	3	1.4276	3
A4	0.6425	4	0.3306	4	0.0359	4	1.4273	4
A5	0.4272	5	0.1320	5	0.0517	5	1.4259	5
A6	0.0712	6	-0.8771	6	0.1192	6	1.4183	6
A7	0.0346	7	-0.9286	7	0.1224	7	1.4181	7

TABLE V. SPEARMAN COEFFICIENT BETWEEN MCDM METHODS IN SCENARIO S1

Weight method	MCDM method	RAWEC	PIV	RAM	Average
ROC	TOPSIS	1	1	1	1.0000
	RAWEC		1	1	
	PIV			1	
RS	TOPSIS	1	0.9643	0.8929	0.9464
	RAWEC		0.9643	0.8929	
	PIV			0.9643	
Lagrange	TOPSIS	1	1	1	1.0000
	RAWEC		1	1	
	PIV			1	

As shown in Table V, the values of the Spearman coefficient are presented for the MCDM methods, wherein the weights of the criteria are determined by ROC, RS, and Lagrange. It is evident that the average value of the Spearman coefficient between the MCDM methods when employing ROC and Lagrange is equivalent to 1. However, when applying the RS, this value is 0.9464, indicating that, in scenario S1, the level of stability in the rank of the alternatives, when using the MCDM methods, is higher when applying the ROC and Lagrange methods, in comparison to the RS method. Table VI presents the ranking of e-bikes for the remaining nine scenarios from S2 to S10, using MCDM with different weighting methods and the calculation of the Spearman coefficient. The rankings of e-bike models are subject to variation in different scenarios and may be affected by the application of different weighting or ranking methodologies in a given scenario.

TABLE VI. THE RANKS OF E-BIKE MODELS IN DIFFERENT SCENARIOS

Scenario	Alt.	ROC				RS				Lagrange			
		TOPSIS	RAWEC	PIV	RAM	TOPSIS	RAWEC	PIV	RAM	TOPSIS	RAWEC	PIV	RAM
S2	A1	1	1	1	1	1	1	1	1	1	2	2	3
	A2	2	3	4	4	2	3	3	5	2	4	4	5
	A3	3	2	2	2	3	2	2	2	3	1	1	1
	A4	4	4	3	3	4	4	4	3	4	3	3	2
	A5	5	5	5	5	5	5	5	4	5	5	5	4
	A6	6	6	6	6	6	6	6	6	6	6	6	6
	A7	7	7	7	7	7	7	7	7	7	7	7	7
S3	A1	1	1	1	1	1	1	1	1	1	1	1	1
	A2	2	2	2	2	2	3	4	2	2	2	2	3
	A3	3	3	3	3	3	2	2	3	3	3	3	2
	A4	4	4	4	4	4	4	3	4	4	4	4	4
	A5	5	5	5	5	5	5	5	5	5	5	5	5
	A6	6	6	6	6	6	6	6	6	6	6	6	6
	A7	7	7	7	7	7	7	7	7	7	7	7	7
S4	A1	1	1	1	1	1	1	1	1	1	1	1	1
	A2	2	2	2	3	2	2	3	4	2	2	2	2
	A3	3	3	3	2	3	3	2	2	3	3	3	3
	A4	4	4	4	4	4	4	4	3	4	4	4	4
	A5	5	5	5	5	5	5	5	5	5	5	5	5
	A6	6	6	6	6	6	6	6	6	6	6	6	6
	A7	7	7	7	7	7	7	7	7	7	7	7	7
S5	A1	1	1	1	1	1	1	1	1	1	1	1	1
	A2	2	2	2	3	2	2	3	3	2	2	2	2
	A3	3	3	3	2	3	3	2	2	3	3	3	3
	A4	4	5	5	5	4	5	4	5	5	5	5	5
	A5	5	4	4	4	5	4	5	4	4	4	4	4
	A6	6	6	6	6	6	6	6	6	6	6	6	6
	A7	7	7	7	7	7	7	7	7	7	7	7	7
S6	A1	3	1	3	3	1	1	1	2	3	3	3	4
	A2	5	5	5	5	4	4	5	5	5	5	5	5
	A3	1	2	1	1	2	2	2	1	1	1	1	1
	A4	2	3	2	2	3	3	3	3	2	2	2	2
	A5	4	4	4	4	5	5	4	4	4	4	4	3
	A6	6	6	6	6	6	6	6	6	6	6	6	6
	A7	7	7	7	7	7	7	7	7	7	7	7	7
S7	A1	1	1	1	1	1	1	1	1	1	1	1	1
	A2	2	2	2	2	2	2	3	3	2	2	2	2
	A3	3	3	3	3	3	3	2	2	3	3	3	3
	A4	4	4	4	4	4	4	4	4	4	4	4	4
	A5	5	5	5	5	5	5	5	5	5	5	5	5
	A6	7	7	7	7	6	7	6	7	7	7	7	7
	A7	6	6	6	6	7	6	7	6	6	6	6	6
S8	A1	1	1	1	1	1	1	1	1	1	1	1	2
	A2	2	2	2	2	2	2	3	3	2	2	2	1
	A3	3	3	3	3	3	3	2	2	3	3	3	3
	A4	4	4	4	4	4	4	4	4	4	4	4	4
	A5	5	5	5	5	5	5	5	5	5	5	5	5
	A6	6	6	6	6	6	6	6	6	6	6	6	6
	A7	7	7	7	7	7	7	7	7	7	7	7	7
S9	A1	1	1	1	1	1	1	1	1	1	1	1	1
	A2	2	2	2	2	2	2	3	3	2	2	2	2
	A3	3	3	3	3	3	3	2	2	3	3	3	3
	A4	4	4	4	4	4	4	4	4	4	4	4	4
	A5	5	5	5	5	5	5	5	5	5	5	5	5
	A6	7	7	7	7	7	7	7	7	7	7	7	7
	A7	6	6	6	6	6	6	6	6	6	6	6	6
S10	A1	2	4	4	5	2	3	4	5	4	5	5	5
	A2	1	1	1	1	1	1	2	2	1	1	1	1
	A3	3	2	2	2	3	2	1	1	2	2	2	2
	A4	4	3	3	3	4	4	3	3	3	3	3	3
	A5	5	5	5	4	5	5	5	4	5	4	4	4
	A6	6	6	6	6	6	6	6	6	6	6	6	6
	A7	7	7	7	7	7	7	7	7	7	7	7	7

However, a thorough examination reveals that alternatives A1, A2, and A3 are ranked low, indicating their prioritization for selection, while alternatives A5, A6, and A7 are ranked high, indicating their non-selection. The findings suggest that alternatives A1, A2, and A3 are the most suitable for selection in the seven e-bike models. Table VII presents a summary of the average value of the Spearman coefficient between the MCDM methods across the ten scenarios performed. In scenario S2, the average value of the Spearman coefficient between the MCDM methods, is the lowest when the Lagrange method is used.

In scenario S10, the average value of the Spearman coefficient between the MCDM methods is minimum when the ROC method is used, while in the remaining eight scenarios, the average value of the Spearman coefficient between the MCDM methods is always the smallest for the case of using the RS method. This finding indicates that, in general, using the RS method to calculate the weights for the criteria makes it difficult to ensure the stability of the rank of the alternatives when ranked by different MCDM methods. In order to calculate weights for criteria by the subjective method, the ROC method or the Lagrange method should be used, while the RS method should not be used.

IV. CONCLUSIONS

Criteria weighting based on their relative importance is referred to as the subjective weighting method. This study evaluated the performance of three subjective weighting methods, Rank Order Centroid (ROC), Rank Sum (RS), and Lagrange in ranking seven types of electric bicycles (e-bikes). It was determined that the ROC and Lagrange methods are suitable for calculating weights for criteria using a subjective approach, while the RS method is not recommended for this purpose. The study's findings indicate that the alternatives A1, A2, and A3 are the preferred ones, among the seven types of e-bikes surveyed, while A5, A6, and A7 are the least preferred. A limitation of the study is the analyses of only three subjective weighting methods, while others left out for future examination. The recommendation of using the ROC or the Lagrange method, over the RS, can only be applied to the evaluation of e-bikes.

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