Hybrid NSGA-II–MARCOS Optimization of a Dual-Input Spur Gearbox: Targeting Efficiency and Minimal Length

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ABSTRACT

This study presents a hybrid optimization framework for the design of a dual-input two-stage helical gearbox, in which the fast stage is split into two parallel power paths to improve load distribution and design flexibility. The design objectives are defined to explicitly reflect two conflicting physical requirements: minimizing the total axial length, which represents structural compactness under spatial constraints, and maximizing mechanical efficiency, which is governed by cumulative mechanical and tribological losses. The optimization process employs the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to generate Pareto-optimal solutions that represent trade-offs between conflicting objectives. To support engineering decision-making among Pareto-optimal configurations, the Multi-Attributive Real Comparative Analysis (MARCOS) method is subsequently integrated to rank candidate designs and identify balanced solutions based on compactness-efficiency compromise. A comprehensive gear system model is developed, incorporating gear geometry, shaft layout, meshing conditions, and detailed loss calculations. The results reveal clear physical trends: increasing the transmission ratio leads to longer gearbox structures and higher mechanical losses, while the dual-input first stage mitigates efficiency deterioration by redistributing torque and reducing localized loading. This approach offers not only algorithmic optimization but also mechanically meaningful design insights, providing valuable insights for engineers facing spatial constraints and high-performance requirements in modern mechanical transmission systems.

Keywords-two-stage gearbox; multi-objective optimization; NSGA-II; gear ratio; face width coefficient; efficiency; volume; design trade-off

I. INTRODUCTION

Helical gearboxes play a crucial role in industrial and automotive power transmission systems, where efficiency, structural compactness, and operational stability are essential. In particular, dual-input or split-input helical gearboxes—where the first stage is divided into two parallel paths—offer advantages in load distribution and symmetry, making them attractive for applications requiring compact, efficient, and high-torque solutions. However, the optimal design of such systems poses a challenge due to the conflicting nature of key objectives, such as minimizing the gearbox length and maximizing efficiency.

Recent research has shown a growing interest in applying Multi-Objective Optimization (MOO) techniques to gearbox design. In [1], the transmission system of an aeroengine accessory gearbox was optimized using a heuristic algorithm to balance noise, efficiency, and cost. Similarly, MOO was used in [2] to reduce transmission error and increase efficiency in gear unit design, highlighting the trade-off between performance and vibrational behavior. In [3], the Non-dominated Sorting Genetic Algorithm III (NSGA-III) was applied to high-speed rotating components to optimize angular contact ball bearings in aircraft gearboxes, confirming the effectiveness of evolutionary algorithms in mechanical system design. Earlier, in [4], a stochastic MOO method was introduced for synchronizer and selector mechanisms, laying the foundation for gearbox subcomponent optimization.

Specific studies on two-stage helical gearboxes include the work in [5], which sought to improve efficiency and reduce casing height using multi-objective approaches. The focus in [6] was on spur gear pairs, demonstrating that optimizing volume and efficiency simultaneously significantly improved performance metrics. In [7], the problem was extended to large-scale marine gearboxes using topology optimization.

Integrated frameworks have been developed to support decision-making among Pareto-optimal solutions. In [8], the Multi-Attributive Real Comparative Analysis (MARCOS) method was applied to optimize a two-stage helical gearbox with double gears in the first stage, demonstrating its effectiveness in selecting balanced designs. In [9], deep learning was used to diagnose gearbox behavior, emphasizing the growing role of intelligent systems in gearbox monitoring and tuning. In [10], the TOPSIS method was adopted for gearbox optimization, achieving improvements in both volume and efficiency. In a related study [11], gear shifting mechanisms were optimized to improve fuel economy and reduce emissions, further supporting the importance of gearbox design in broader energy systems.

Uncertainty in load and operating conditions presents another major challenge in gearbox design. In [12], a robust optimization strategy was proposed to handle such uncertainties, while in [13], tribological constraints were incorporated into wind turbine gearboxes. In [14], layered optimization techniques were applied to a magnetic planetary gear, confirming the flexibility of modern MOO approaches.

Theoretical foundations for evolutionary algorithms in MOO have been well documented in [15], which continues to guide algorithm development, such as NSGA-II, which is widely adopted for its convergence and diversity preservation capabilities. In [16], NSGA-II was applied to spur gearboxes, achieving balanced trade-offs between mass and efficiency. In [17], this approach was extended to planetary gear trains under uncertainty, confirming its robustness.

Recent studies on gearbox optimization increasingly emphasize the trade-off between geometric compactness and mechanical efficiency, particularly in multi-stage transmissions. In [6], it was shown that minimizing gearbox volume or axial dimensions often leads to increased contact loads and frictional losses, resulting in efficiency degradation. These results highlight that geometric parameters such as center distance and face width directly influence both structural size and loss mechanisms, making compact gearbox design inherently a multi-objective problem rather than a purely geometric one.

From a tribological perspective, gearbox efficiency is closely related to frictional behavior at contacting interfaces, especially during gear meshing. Experimental investigations in [18] showed that gear material combinations and lubrication conditions significantly affect the coefficient of friction and, consequently, power losses during operation. Further studies incorporating advanced lubricant additives, such as carbon nanotubes, revealed that friction reduction can improve efficiency and vibration behavior, but often requires careful control of contact conditions and load distribution [19]. These findings underscore that efficiency optimization cannot be decoupled from tribological and material considerations.

According to the classical gear drive theory [20], the mechanical efficiency of a gearbox is governed by energy dissipation mechanisms, including losses in gear meshing, bearings, seals, and idle motion. These losses increase with transmitted load, contact pressure, rotational speed, and unfavorable lubrication conditions, making friction at the tooth contact and bearing interfaces the dominant contributors to efficiency degradation in multi-stage gear systems. Design strategies aimed at reducing frictional losses—such as improving load sharing or reducing local contact stresses—typically require increased face width, larger center distance, or more complex transmission layouts, which inevitably enlarge gearbox dimensions. Consequently, gearbox design involves an intrinsic trade-off between structural compactness and energy dissipation, especially pronounced in high-ratio and multi-stage configurations [20].

There are more than 200 Multi-Criteria Decision-Making (MCDM) methods available, and the rankings of alternative choices differ considerably when employing different methods [21]. Some studies use a single MCDM method, while others use multiple methods simultaneously. However, in most studies, in addition to using an MCDM method to rank options, additional methods are always needed to calculate the weight of the criteria [22]. Among MCDM methods, MARCOS stands out because it seamlessly integrates with various data normalization methods and reduces the reliance on predefined weights for criteria when identifying the optimal option [23].

In this context, the optimization of dual-input or split-path gearbox architectures offers a promising approach to mitigate this trade-off. By redistributing the torque among parallel power paths, such configurations can reduce localized contact loads and associated frictional losses while maintaining acceptable efficiency levels. However, the resulting geometric complexity necessitates a systematic MOO framework capable of capturing the coupled effects of geometry, load distribution, and energy dissipation. These considerations provided the physical motivation for this study, in which gearbox length is treated as a geometric compactness objective and efficiency as an energy-based objective within an MOO framework.

Despite these advances, limited work has addressed the optimization of dual-input two-stage helical gearboxes targeting axial length reduction and efficiency enhancement, especially under a hybrid NSGA-II–MARCOS framework. This study fills this gap by combining NSGA-II to explore the Pareto-optimal space and MARCOS to select the best compromise solution based on the designer's preferences. The goal is to provide a reliable and compact transmission design suitable for high-performance applications where spatial constraints and energy losses must be carefully balanced.

II. OPTIMIZATION PROBLEM

A. Gearbox Length Calculation

The length of the gearbox L_{gb} can be calculated by (Figure 1):

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \quad (1)$$

where $\delta = 7 \div 10$ (mm); d_{w1i} and d_{w2i} $(i = 1 \div 2)$ denote the pitch diameter of the pinion and the gear of the first and the second stage, which can be computed by [24]:

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \quad (2)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \quad (3)$$

where a_{wi} $(i = 1 \div 2)$ is the gearbox center distance of stage i, determined by [24]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (4)$$

with T_{1i} $(i = 1 \div 2)$ being the torque on the pinion of i^{th} stage, which can be calculated by:

$$T_{11} = T_{out}/\left(2 \cdot u_{gb} \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \quad (5)$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \quad (6)$$

B. Gearbox Efficiency Calculation

The mechanical efficiency η_{gb} of the gearbox is calculated by accounting for all major sources of power loss:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (7)$$

where P_l is the total power loss in the gearbox, given by [20]:

$$P_l = P_{lg} + P_{lb} + P_{Z0} + P_{ls} \quad (8)$$

where P_{lg}, P_{lb}, P_{ls}, and P_{Z0} are the power losses in the gears, bearings, the idle motion, and seals, which can be computed as in [25].

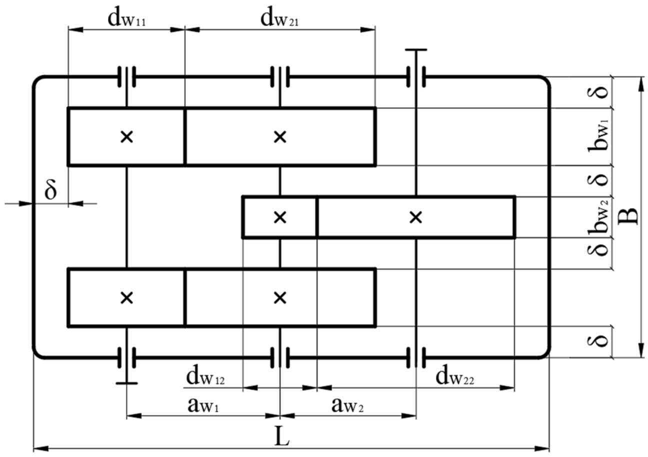

Fig. 1. Schema for gearbox length calculation.

C. Objective Functions and Constraints

The gearbox design problem is formulated as a bi-objective optimization, with two conflicting goals:

- Objective 1: Minimize the total gearbox length L_{gb},
- Objective 2: Maximize mechanical efficiency η_{gb}.

Since both objectives are handled in a minimization framework for NSGA-II, the second objective is reformulated as minimizing the negative efficiency:

$$\min f_1(X) = L_{gb} \quad (9)$$

$$min f_2 = -\eta_{gb} \quad (10)$$

1) Design Variables

Three main design variables are considered: u_1 is the transmission ratio of the first (split) stage, X_{ba1} is the face width coefficient for the first stage, and X_{ba2} is the face width coefficient for the second stage.

2) Design Constraints

$$1 \leq u_i \leq 9 \quad (11)$$

$$0.25 \leq Xba_i \leq 0.4 \quad (12)$$

The optimization process uses NSGA-II to explore the design space and generate a Pareto front, followed by the MARCOS method to evaluate and rank the Pareto-optimal solutions based on user-defined weights and performance preferences.

III. METHODOLOGY

A. NSGA-II Method

NSGA-II is employed to solve the bi-objective optimization problem of minimizing gearbox length and maximizing efficiency. NSGA-II is a robust and well-established multi-objective evolutionary algorithm that effectively generates a diverse set of Pareto-optimal solutions [15]. The workflow of NSGA-II includes:

- Initialization: A population of candidate solutions is randomly generated, each representing a possible gearbox configuration.
- Non-dominated Sorting: Individuals are ranked into Pareto fronts based on dominance relationships.
- Crowding Distance Assignment: Each individual is assigned a crowding distance to preserve diversity within each front.
- Selection, Crossover, Mutation: Binary tournament selection is used to generate offspring through Simulated Binary Crossover (SBX) and polynomial mutation.
- Elitism and Replacement: The next generation is formed by combining parent and offspring populations, selecting the best individuals according to Pareto rank and diversity.

After several generations, NSGA-II outputs a set of non-dominated solutions that represent the trade-off frontier between gearbox length and efficiency.

B. *MARCOS Method*

The MARCOS method is employed to support MCDM by ranking the Pareto-optimal solutions obtained from NSGA-II. MARCOS evaluates each alternative based on its relative closeness to both the ideal solution (AI) and the anti-ideal solution (AAI). To apply this method, the following steps are followed, as described in [26]:

- Step 1: Constructing the initial decision-making matrix:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \quad (13)$$

where m and n are the number of alternatives and the criteria.

- Step 2: Extending the matrix with Ideal and Anti-Ideal solutions

$$X = \begin{array}{c} AAI \\ A_1 \\ A_2 \\ \vdots \\ A_m \\ AI \end{array} \begin{bmatrix} x_{aa1} & \cdots & x_{aan} \\ x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{mn} \\ x_{ai1} & \cdots & x_{ain} \end{bmatrix} \quad (14)$$

where $AAI = min\,(x_{ij})$ and $AI = max\,(x_{ij})$ for the efficiency (benefit criterion), and $AAI = max\,(x_{ij})$ and $AI = min\,(x_{ij})$ for the gearbox length (cost criterion), with $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.

- Step 3: Normalizing the Extended Matrix. The normalized matrix can be calculated by:

$$u_{ij} = x_{AI} / x_{ij} \quad (15)$$

$$u_{ij} = x_{ij} / x_{AI} \quad (16)$$

where (3) is used for cost-type criteria (e.g., length), and (4) is applied for benefit-type criteria (e.g., efficiency).

- Step 4: Weighted normalized matrix. Weights w_j for each criterion are applied to form the weighted matrix $C = \left[c_{ij}\right]_{m\times n}$:

$$c_{ij} = u_{ij} \cdot w_j \quad (17)$$

- Step 5: Computing the utility of alternatives K_i^- and K_i^+ by:

$$K_i^- = S_i / S_{AAI} \quad (18)$$

$$K_i^+ = S_i / S_{AI} \quad (19)$$

where S_i can be found by:

$$S_i = \sum_{i=1}^{m} c_{ij} \quad (20)$$

- Step 6: Calculating the utility function $f(K_i)$ of alternatives by:

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1-f(K_i^+)}{f(K_i^+)} + \frac{1-f(K_i^-)}{f(K_i^-)}} \quad (21)$$

where $f(K_i^-)$ is the utility function linked with the anti-ideal solution and $f(K_i^+)$ is the utility function connected with the ideal solution. These elements can be found by:

$$f(K_i^-) = K_i^+ / (K_i^+ + K_i^i) \quad (22)$$

$$f(K_i^+) = K_i^- / (K_i^+ + K_i^i) \quad (23)$$

- Step 7: Ranking alternatives by maximizing $f(K_i)$.

IV. RESULTS AND DISCUSSION

A. *Relationship Between Transmission Ratio and Stage 1 Gear Ratio*

Figure 2 shows the regression between the overall transmission ratio u_h and the average first-stage gear ratio u_1 across Pareto solutions. The derived linear equation is (R^2=0.9764):

$$u_1 = 1.5650 + 0.1607 \cdot u_h \quad (24)$$

Figure 3 compares this with the linear trend of MARCOS-selected optimal values, yielding an R^2 of 0.9865:

$$u_1 = 1.7740 + 0.1336 \cdot u_h \quad (25)$$

This suggests that MARCOS tends to select configurations with slightly lower slope, balancing compactness and performance by avoiding overemphasis on the first stage.

B. *Variation of Mean Gearbox Length and Efficiency*

Figure 4 illustrates how the mean gearbox length and the mean efficiency vary with u_h. The key observations are:

- •Mean length increases steadily from ~430 mm to ~457 mm as u_h grows, reflecting the geometric expansion needed to achieve higher reduction.
- Mean efficiency decreases from ~95.5% to below 88%, indicating increased losses due to gear meshing and bearing friction in more complex configurations.

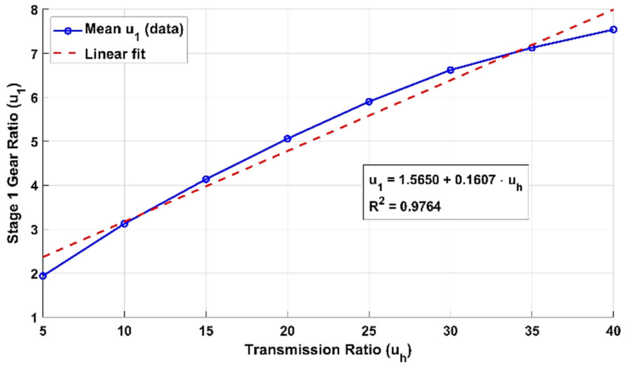

Fig. 2. Linear regression of mean u_1 vs. u_h from NSGA-II Pareto-optimal solutions.

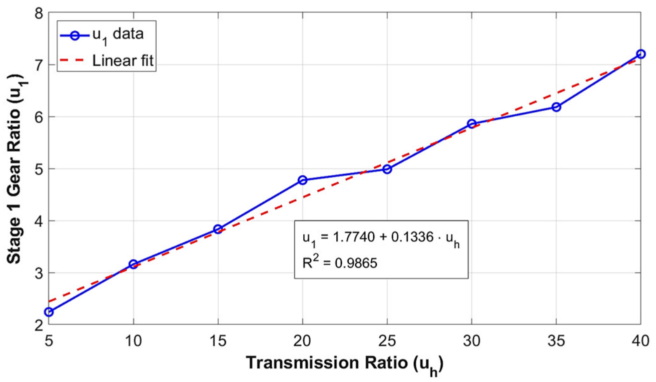

Fig. 3. Linear regression of MARCOS-selected optimal u_1 vs. u_h.

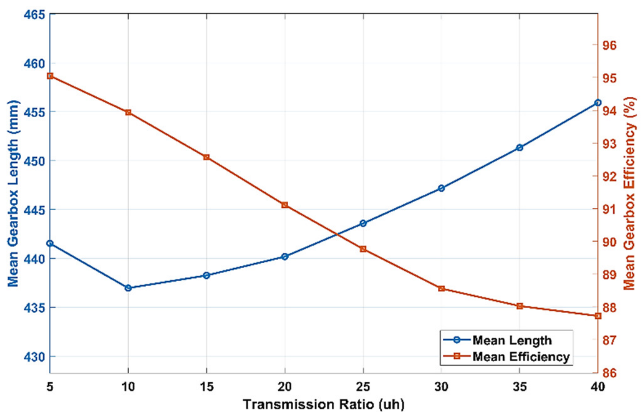

Fig. 4. Trend of mean volume and efficiency vs. gearbox ratio (u_h).

As illustrated in Figure 4, the mean gearbox efficiency decreases as the overall transmission ratio u_h increases, whereas the mean gearbox length shows an increasing trend. This can be explained by the rise in transmitted torque at higher gear ratios, especially in the low-speed stage, which leads to increased contact forces, intensified sliding, and higher frictional losses in the gear mesh, as well as increased bearing losses due to larger reaction forces. Consequently, the cumulative mechanical losses increase, resulting in a reduction in efficiency with increasing gear ratio, in agreement with classical gear transmission theory [20]. At the same time, larger gear ratios require increased geometric dimensions to sustain higher torque levels, leading to a longer gearbox. This opposing trend reaffirms the trade-off between performance and size in gearbox design.

C. Pareto Fronts of Gearbox Length vs. Efficiency

Figure 5 presents the Pareto fronts for each u_h in the efficiency-length objective space.

- Lower values of u_h (e.g., 5–15) yield high-efficiency, short-length designs.
- As u_h increases, the Pareto front shifts downward and rightward, showing reduced performance and increased structural demand.

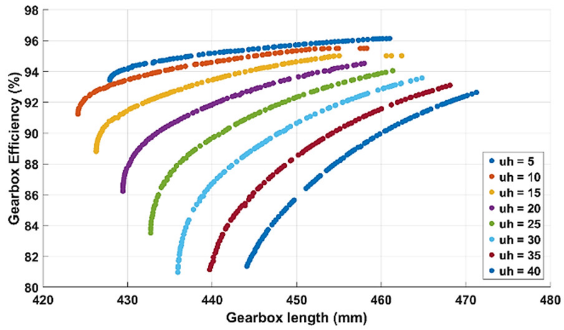

Fig. 5. Pareto fronts of gearbox efficiency versus length for different u_h.

This behavior illustrates the strength of NSGA-II in capturing the full spectrum of trade-offs and the necessity of applying MCDM tools like MARCOS to choose feasible designs in practical scenarios.

D. Evaluation of Optimal Solutions Using MARCOS

Table I summarizes the best solutions obtained from NSGA-II for each value of u_h, selected based on the optimal trade-off between compactness and efficiency. As can be observed, u_1 increases progressively with u_h, aligning with the regression trend in Figure 4. The face width coefficients (X_{ba1} and X_{ba2}) remain close to their lower bounds, indicating their marginal impact within the feasible design space. Efficiency consistently decreases while volume increases with u_h, validating the trade-off behavior seen in the Pareto distribution.

Table I summarizes the MARCOS-selected optimal designs for each transmission ratio u_h, showing that:

- Gearbox length increases gradually with u_h,
- Efficiency decreases slightly, but remains above 89% even at u_h =40,
- X_{ba2} remains fixed at 0.40, indicating the second stage consistently operates under higher torque and requires wider gears,
- X_{ba1} varies, with higher values (0.34–0.40) at low u_h, and smaller values (~0.28–0.31) at high u_h, suggesting a design tendency to minimize bulk in the split first stage under an increased overall ratio.

TABLE I. OPTIMAL GEARBOX CONFIGURATIONS SELECTED BY THE MARCOS METHOD

u_h	u_1	X_{ba1}	X_{ba2}	Length (mm)	η_{gb} (%)
5	2.24	0.40	0.40	429.22	94.06
10	3.16	0.40	0.40	427.95	93.12
15	3.83	0.34	0.40	438.76	93.28
20	4.78	0.36	0.40	439.05	91.64
25	4.99	0.28	0.40	455.70	93.32
30	5.86	0.30	0.40	454.54	91.80
35	6.18	0.31	0.40	457.37	90.92
40	7.20	0.31	0.40	459.83	89.94

E. Sensitivity Analysis of MARCOS-Selected Solutions

To further evaluate the robustness of the optimal gearbox designs selected by the MARCOS method, a sensitivity analysis was conducted with respect to the key design variables, namely the first-stage transmission ratio u_1, the face width coefficient of the first stage (X_{ba1}), and the face width coefficient of the second stage (X_{ba2}). The analysis was performed using a One-At-a-Time (OAT) perturbation approach, in which each variable was independently varied within ±5% and ±10% around the MARCOS-selected optimal solutions, while the remaining variables were kept constant.

Figure 6(a) presents the tornado plot illustrating the sensitivity of gearbox efficiency to variations in the design variables. It can be observed that the efficiency is primarily influenced by u_1 and X_{ba1}, both of which lead to the largest maximum variations in efficiency. This behavior is physically consistent, as u_1 governs the distribution of the transmission ratio between stages, directly affecting sliding velocity and load conditions in the high-speed stage, while X_{ba1} controls the face width of the first-stage gears, where frictional and meshing losses are most pronounced. In contrast, the influence of X_{ba2} on efficiency is negligible, indicating that variations in the face width of the low-speed stage have only a minor effect on the overall power losses.

Figure 6(b) shows the tornado plot corresponding to the sensitivity of gearbox length. In this case, X_{ba2} emerges as the dominant factor affecting the total gearbox length, whereas u_1 and X_{ba1} exhibit only moderate influence. This result can be attributed to the geometric contribution of the second-stage gears to the overall axial layout of the gearbox. Since the low-speed stage typically involves larger gear dimensions, variations in X_{ba2} directly translate into changes in the gearbox footprint, particularly in terms of axial length.

Overall, the sensitivity results reveal a clear separation between efficiency-driven and geometry-driven influences. While u_1 and X_{ba1} predominantly govern efficiency-related behavior, X_{ba2} mainly controls structural compactness. Importantly, the maximum variations in both gearbox length and efficiency remain within a limited range, confirming that the MARCOS-selected solutions are not excessively sensitive to moderate parameter perturbations. This demonstrates the robustness of the proposed NSGA-II-MARCOS optimization framework and supports the reliability of the selected optimal designs for practical gearbox applications.

(a)

(b)

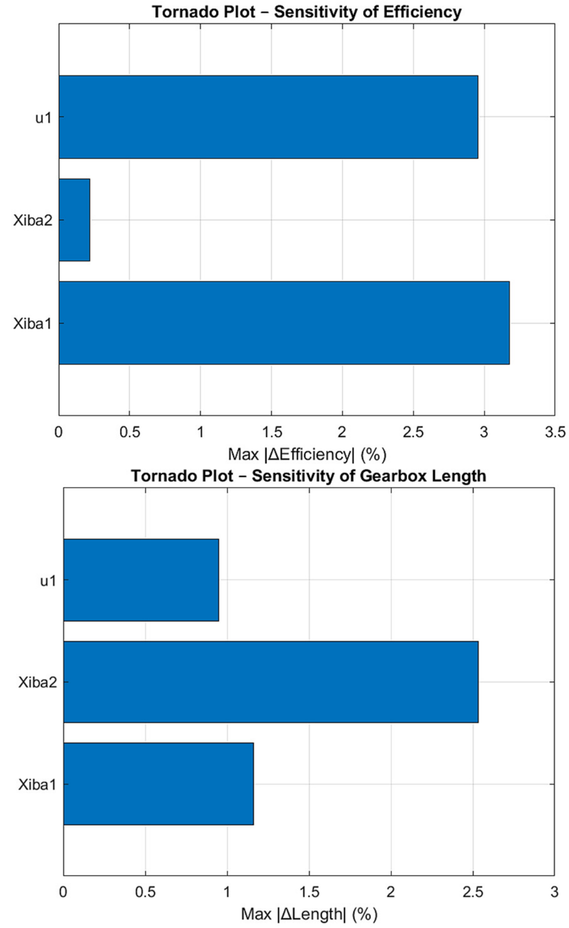

Fig. 6. Tornado plots of sensitivity for (a) gearbox efficiency and (b) gearbox length.

V. CONCLUSIONS

This study presented a hybrid MOO framework for the design of a dual-input two-stage helical gearbox, where the primary objectives were to minimize the overall axial length and to maximize transmission efficiency. This approach combined the global search capability of the NSGA-II algorithm with the structured decision-making of the MARCOS method, enabling both comprehensive Pareto-front generation and rational solution selection.

A detailed computational model was developed to evaluate gear geometry, meshing conditions, casing configuration, and mechanical losses. Through this model, NSGA-II successfully produced diverse sets of Pareto-optimal solutions for a range of transmission ratios u_h. Analysis of these results revealed consistent trends: increasing u_h led to greater gearbox length and reduced efficiency, confirming the structural-performance trade-off in compact power transmission systems. The MARCOS method was then applied to rank the Pareto-optimal solutions. It demonstrated the ability to consistently select configurations that strike a practical balance between compactness and performance. The selected designs showed gradual increases in u_1 and casing length, while maintaining high efficiency across all u_h values.

The optimization results obtained confirm the initial expectations and assumptions of this study. Specifically, the analysis consistently shows that increasing the overall gear ratio leads to a reduction in gearbox efficiency and an increase in gearbox length, reflecting the anticipated trade-off between efficiency and structural compactness. No opposite or contradictory trends were observed within the investigated design space, indicating that the adopted modeling assumptions and the hybrid NSGA-II–MARCOS framework provide physically consistent and reliable results for system-level gearbox design.

Nevertheless, this present study is limited to a deterministic, steady-state modeling framework based on nominal gear geometry. For detailed design and validation, optimized configurations should be further examined using finite element analyses (FEM) to account for elastic deformations, contact stress distributions, and housing stiffness. In addition, the influence of dynamic and variable loading conditions, as well as the integration of experimental efficiency data, would be necessary to extend the applicability of the proposed approach toward high-fidelity performance prediction under real operating conditions.

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