The Effects of Superstatistics Properties on Hot Plasma

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Abstract-The electron impact ionization is a crucial atomic process in the collisional radiative model and the study of ionization balance. The superstatistics theory, which was originally proposed for the study of non-equilibrium complex systems, has recently been extended to studies of small systems interacting with a finite environment due to their interesting statistical behavior. This paper introduces the superstatistics formalism in the case of ionization rates with different values of the dynamical parameter *q* and shows how it affects the calculation of the ionization rates for Li^+. Moreover, the distribution function for the effective Boltzmann factor of superstatistics was swapped.

Keywords-ionization rates; Boltzmann factor; superstatistics; code FAC; cross-section

I. INTRODUCTION

Electron impact ionization of neutral atoms has been proven to be significant in a variety of fields, including mass spectrometry and plasma physics [1]. Thomson utilized the ionization of atoms by charge impact in the early 1910s. Since then, many others, have amended it [2, 3]. Furthermore, electron-impact ionization has been used to measure a detailed ionization spectrum and cross-sections using various methods [4]. Plasma is important in nuclear physics because many studies showed that it can take various shapes, including non-equilibrium plasma that is used to change the characteristics of the surface [5, 6]. The characteristics of non-equilibrium plasmas are known through the ionization of atoms and molecules by electron impact, which has great practical significance. Experimental data, which include laboratory plasma at low and high temperatures, atmospheric physics, and mass spectrometry, are also significant as they can be compared to theoretical predictions [7, 8].

Ionization occurs when an electron collides with a neutral particle with sufficient energy to produce a positive ion and a free electron [9]. Superstatistics is a subfield of statistical physics or mechanics that studies nonlinear and non-equilibrium systems [10]. It is characterized by the use of a superposition of several distinct statistical models to attain the desired nonlinearity. In ordinary statistical terms, this is similar to compounding the distributions of random variables, and it can be thought of as a simple case of a doubly stochastic model [11, 12]. Furthermore, superstatistics contends that in systems with temperature fluctuations, coarse-grained measurements of the energy performed over spatial and temporal scales are larger than those determined through the correlation properties of the temperature.

Yield statistical distributions can be considered as a superposition of canonical distributions. Superstatistics [10, 13-17] is a powerful modeling and/or analysis technique for complex systems with two or more distinct time scales in dynamics. The fundamental idea is to consider a superposition of several systems in local equilibrium, each with its inverse temperature, and then take an average over the fluctuating that are disseminated according to some probability density for theoretical modeling. In general, the parameter does not have to be an inverse temperature but could be any system parameter that shows large-scale fluctuations, such as energy dissipation in a turbulent flow or financial market volatility [18]. Finally, all relevant expectation values for the complex system are averaged over this distribution. Several applications have been previously described, including modeling the statistics of classical turbulent flow [19, 20], quantum turbulence [21], spacetime granularity [22], stock price changes [23, 24], virus infection pathways [25], and more [4, 25-27].

Once incorporated into the fluctuating parameter, superstatistical systems are efficiently described by more general measures than the Boltzmann–Gibbs entropy. The following question can be asked: What entropy statistics appear in the case of a fluctuating control parameter? This study aimed to develop a generalization of the Beck-Cohen superstatistics that makes possible to appropriately consider such fluctuations. The main idea was to use superstatistics to replace the distribution function via the effective Boltzmann factor and introduce the superstatistics formalism in the case of ionization rates with different values of *q*.

II. SUPERSTATISTICS

Long-range interactions between different ions and electrons dominate in the plasma state, causing these systems to

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be in highly non-equilibrium states. Even when these systems achieve a steady state, they do not adhere to Boltzmann-Gibbs statistics [14]. The following critical question is asked: Do these steady-state plasmas have a well-determined temperature? Superstatistics was introduced in 2003 as an interesting suggestion for the treatment of non-equilibrium steady-state systems [10] and depends on ordinary Boltzmann statistics, which are formed as a tool to study systems with complex dynamics and non-equilibrium [15]. In a non-Boltzmann-Gibbs distribution, where the systems are out of equilibrium and non-extensive, superstatistics displays a relative motion to explain the gain of it [27]. These statistics admit the superposition of canonical ensembles at different temperatures [28]. Superstatistics is a branch of statistical mechanics that occurs in non-equilibrium and stable states with large parameter fluctuations. They were named superstatistics because they are a type of "statistics of statistics" [28]. Superstatistics systems are a superposition of two (or more) different statistics: one is given by ordinary Boltzmann factor, and the other by large-scale fluctuations of one (or many) intensive parameters, e.g. the inverse temperature [29]. Since then, it has been used to describe a wide variety of dynamic structures with changing environmental conditions [30]. This new type relies on ordinary statistical mechanics, described by the ordinary Boltzmann factor $e^{-\beta E}$, thus an effective Boltzmann factor $B(E)$ for the full system can be defined as [31]:

$$B(E) = \int_0^\infty f(\beta) e^{-\beta E} d\beta = \langle e^{-\beta E} \rangle \quad (1)$$

where $f(\beta)$ is the normalized probability distribution, E is the effective energy in each cell, and $\beta = \frac{1}{K_B T}$ is approximately a constant. At the same time, the normalized probability distribution must be realized by probability density [10]:

$$f(\beta) = \delta(\beta - \beta_0) \quad (2)$$

where β_0 is the average of β (fixed constant) and δ is the Dirac delta. The probability density $f(\beta)$ has to provide the following conditions:

1. It must be a normalized probability density, it may be a physically relevant density from statistics, such as Gaussian, uniform, etc. [32].
2. When there are no fluctuations in intensive quantities, the new statistics decrease to Boltzmann-Gibbs statistics [33, 34], and has to be normalizable [34].

There are two types of superstatistics:

- Type-A superstatistics, where the Boltzmann factor is normalized to yield the stationary long-term probability distribution [35]:

$$P(E) = \frac{1}{Z(\beta)} B(E) e^{-\beta E} \quad (3)$$

- Type-B superstatistics: As the distribution in type A is not properly normalized, a better way of writing would be [36]:

$$P(E) = \int \frac{e^{-\beta E}}{Z(\beta)} f(E) d\beta \quad (4)$$

$$Z(\beta) = \int_0^\infty B(E) dE \quad (5)$$

where $Z(\beta)$ is the normalization constant of $e^{-\beta E}$ for a given β.

This approximation represents the leading order correction to ordinary statistical mechanics for a nonhomogeneous system with small temperature fluctuations. Following this approximation, it was demonstrated that the generalized Boltzmann factor for any distribution is given by the following:

$$B(E) = e^{-\beta_0 E}\left(1 + \frac{1}{2}\sigma^2 E^2 + \sum_{r=3}^{\propto} \frac{(1)^r}{r!} \langle (\beta - \beta_0)^r \rangle E^r\right) \quad (6)$$

with average $\langle \beta \rangle = \beta_0 = \int_0^\infty \beta f(\beta)\, d\beta$, and variance $\sigma^2 = \langle \beta^2 \rangle - \langle \beta \rangle^2$. The zeroth-order approximation to $B(E)$ corresponds, as is expected, to the "pure" Boltzmann statistic: $B(E_n) \sim e^{-\langle \beta \rangle E_n}$. This universality indicated that the superstatistics theory is restricted by two universal parameters (β, q) with:

$$q = \frac{\langle \beta^2 \rangle}{\langle \beta \rangle^2} \quad (7)$$

In (7), the parameter q is simply the coefficient of variation of the distribution $f(\beta)$ defined by the standard deviation to mean ratio. The fluctuation of the intensive parameter β is controlled by this parameter. If there are no fluctuations, q=1 [10]. To easier deal with the Boltzmann factor, it was imposed into a general form as [10]:

$$B(E) = e^{-\beta_0 E}\left(1 + \frac{1}{2}(q-1)\beta_0^2 E^2 + g(q)\beta_0^3 E^3 \ldots\right) \quad (8)$$

where the function $g(q)$ is determined by the used superstatistics. The following can be obtained:

$$g(q) = 0 \quad \text{(uniform and 2-level)} \quad (9)$$

$$= -\frac{1}{3}(q-1)^2 \quad \text{(Gamma)} \quad (10)$$

$$= -\frac{1}{3}(q^3 - 3q + 2) \quad \text{(log-normal)} \quad (11)$$

$$= -\frac{1}{3}\frac{(q-1)(5q-6)}{3-q} \quad \text{(F-distribution)} \quad (12)$$

Finally, superstatistics has become a hot topic in recent years, with applications in a variety of fields of physics [37].

III. Calculation of the Ionization Rates from Superstatistics Distributions

This study aims to calculate the ionization rates from superstatistics distributions. Free electrons are represented by a specific energy distribution within the plasma. The ionization rates were calculated through the integration of a cross-section of a collision that depends on energy over the energy distribution functions. The ionization rates by electron impact were calculated by averaging the product of the electron's velocity and the ionization cross-section. An ionizing collision results in an electron impact process, resulting in a low-energy electron and an atomic ion if an electron receives sufficient energy (via super-elastic collisions), which is equivalent to the neutral atom's ionizing potential. The ionization rates for direct ionization are defined by [38, 39]:

$$\tau = \int v\sigma(\varepsilon) F(\varepsilon) d\varepsilon \quad (13)$$

where v is the velocity of the incident electron, $\sigma(\varepsilon)$ is the impact ionization cross sections calculated by the FAC code [40, 41], $F(\varepsilon)$ is the electron energy distribution function, and ε is the energy of the impacting electron.

The associated rate coefficients could be represented by a simple analytical formula using the appropriate approximation for the energy dependency of the cross-sections. Two types of distribution functions were used in previous studies for the ionization rates: Maxwellian and non-Maxwellian. Indeed, superstatistics provides an effective description of their tools, having demonstrated their adaptability in advanced research. As a result, the calculations were extended to the case of plasma by replacing $F(\varepsilon)$ in (13) with the effective Boltzmann factor $B(E)$ using the superstatistics distribution function. The FAC code [40] was used for the calculation of the Li+ cross-sections, where the relativistic Distorted Wave (DW) approximation method with an interpolation-factorization method [42] was used. The type of most physically pertinent averaging process for a generic system with intense parameter fluctuations is an intriguing question. The answer to this question depends on the physical problem under consideration. Depending on the underlying microscopic dynamics, different non-equilibrium systems may necessitate different types of superstatistics. When using the effective Boltzmann factor:

$$\tau = \int v\sigma(E)B(E)dE \qquad (14)$$

the ionization rates become:

$$\tau = \int_0^{\infty} v\sigma(E)e^{-\beta_0 E}\left(1+\frac{1}{2}(q-1)\beta_0^2E^2+g(q)\beta_0^3E^3\right)d\,E \qquad (15)$$

where $g(q)$ refers to the previously shown superstatistics formulas.

IV. Results and Discussion

The above definitions can be used to obtain the results of measuring the ionization rates of Li^+ for the Gamma, log-normal, and F-distributions for various values of q in Figures 1, 2, and 3. Li^+ ionization rates were calculated using (15) and the various $g(q)$ functions, which rely on the superstatistics concept. The values of the rate coefficients in Figures 1-3 increase very quickly at low temperatures. In the case of high temperatures, the ionization rate curves were very sensitive to q values, and the curves gradually moved away from each other as the q values increased. There is a convergence of the curves for the same distribution and a lot of coordination between the curves for all three functions, especially for low values of q.

In general, the behavior of all superstatistics at high temperatures differs, as it is strongly dependent on the $f(\beta)$ function, but the behavior at low temperatures is universal. Furthermore, it should be noted that the first-order corrections to the Boltzmann factor $e^{-\beta_0 E}$ for the above distribution functions can be written in a universal form. The physical meaning of this broadly defined parameter q is simply the coefficient of variation of the distribution $f(\beta)$, defined by the standard deviation/mean ratio. If there are no fluctuations in β, q equals to 1.

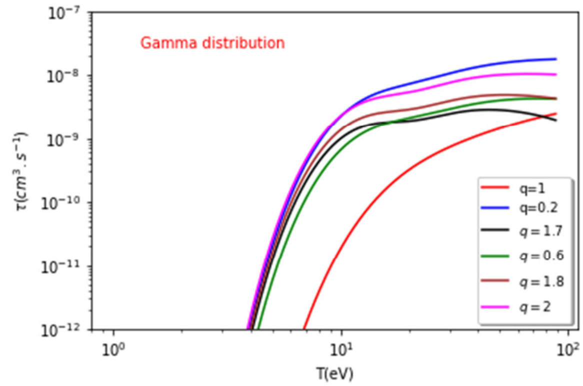

Fig. 1. Ionization rates of Li^+ obtained by the Gamma distribution with different values of q.

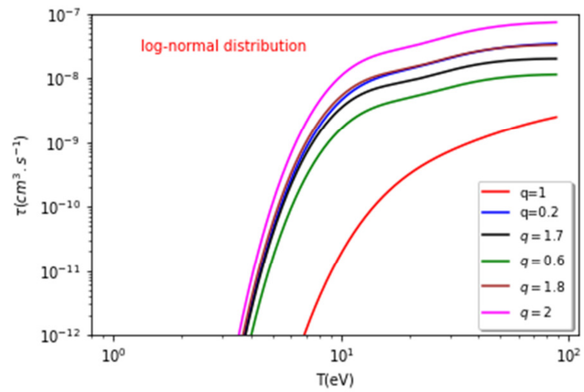

Fig. 2. Ionization rates of Li^+ obtained by the log-normal distribution with different values of q.

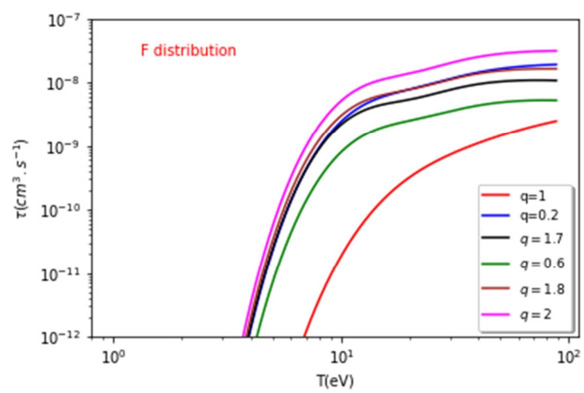

Fig. 3. Ionization rates of Li^+ obtained by the F-distribution with different values of q.

V. Conclusion

This study used the FAC code for the calculation of the cross-sections of Li^+. The Maxwellian distribution function of energy was replaced by an effective Boltzmann factor for various distribution functions to estimate ionization rates from cross-sections. As superstatistics and Maxwellian statistics have similarities when q is close to 1, the results showed a good agreement between the designed curves for various values of q. However, instead of applying deviations to a superstatistical non-equilibrium system, they were applied to an ordinary statistical mechanics equilibrium system with an inverse temperature. Moreover, superstatistics theory requires the presence of temporally local equilibrium within the plasma, which appears to be a non-equilibrium thermodynamic system. Thus, the introduction of superstatistics formalism in the case of ionization rates with different values of q was limited to agreement with other statistics. The connection between the Maxwellian distributions and the Beck-Cohen superstatistics can be noted, as the relationship between the superstatistics approach and the different values of q on the calculation of lithium ionization rates Li^+ was shown.

The dynamic parameter q of (7) can be defined in superstatistics methods. It was shown that all superstatistics have universal behavior for small variances in the fluctuations. There were differences of large variance that gave information on the underlying complex dynamics [43]. In general, different types of superstatistics may be required for complex non-equilibrium problems. The Tsallis statistics are just one example of many new statistical methods that could be used [36]. There is no reason to believe that other superstatistics would not exist in nature.

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