The Application of LQG Balanced Truncation Algorithm to the Digital Filter Design Problem

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Abstract-This paper presents a method for using a model reduction algorithm to design low-order digital filters. Designing an IIR digital filter that meets the specifications often leads to a high-order digital filter. To reduce the computation time and increase the response rate of the filter, we need to reduce the order of the high-order digital filter. Applying the LQG balanced truncation algorithm to reduce the demand for high-order digital filters shows that low-order filters can completely replace high-order digital filters. The simulation results show that the use of the LQG balanced truncation algorithm in order to reduce the filter order is correct and efficient.

Keywords-model order reduction algorithm; LQG balanced truncation algorithm; digital filter

I. INTRODUCTION

A filter is a circuit that removes or "filters out" a specified range of frequencies. In other words, it decomposes the spectrum of the signal into frequency components that will pass and frequency components that will be suppressed. Analog filters use analog electronic circuits made up of components such as resistors, capacitors, and optical amplifiers to produce the required filtering effect. A digital filter uses a digital processor to perform numerical calculations on the sampled values of the signal [1-7]. The processor can be a general-purpose computer such as a PC or a dedicated DSP (Digital Signal Processor) chip. We know that during digital signal processing, the bandwidth of the frequency band can be varied as filters will suppress unwanted frequency components and the bandwidth of the processed signal will decrease and we can reduce the sampling frequency to match the signal's spectral width, which will reduce the number of computations in the digital filter.

Compared with analog filters, digital filters have the following advantages: it is easy to change the filter structure without affecting the hardware, they are easy to design, deploy, and test, digital filters are extremely stable with time and temperature, they have high signal processing flexibility, and they can handle both low-pass and high-pass filtering accurately [1-2]. Due to the superior properties of digital filters, the application of digital filters is increasingly being expanded in the fields of telecommunications, speech processing, image processing, antenna systems, digital audio engineering, and multiplexing systems.

Filter design involves constructing a filter transfer function that satisfies a given frequency response. To design a digital filter, there are two basic ways:

- Method 1: Design an analog filter that satisfies the given requirements and then apply the equivalent transformation to form a digital filter.
- Method 2: Apply computer-aided optimization methods to determine whether a digital filter satisfies the requirements.

Of these two designs, the first is commonly applied due to its ease of implementation. The second method is highly complex, so it is rarely used. To design an analog filter, it is necessary to solve two problems: The first is to determine the main requirements of the filter to be designed. The second is to determine whether the filter is designed to be FIR (Finite-Impulse Response) or IIR (Infinite-Impulse Response.)

- Design of the FIR filter: the output is the impulse response vector $h = [h_0, h_1, h_2, \ldots, h_N]$.
- Design an IIR filter: the output is the coefficient vectors of the numerator and denominator of the transfer function $b = [b_0, b_1, \ldots, b_N]$ and $a = [1, a_1, a_2, \ldots, a_N]$.

The FIR filter has the advantage of a stable, linear phase characteristic. Its disadvantage is that when a large frequency response is desired, the large filter length N increases computational cost. The IIR filter has the advantages of low computational cost and efficient implementation in cascade of 2^{nd}-order circuits. The disadvantages of the IIR filter are that there is instability due to the quantization of the coefficients which can push the poles outside the unit circle and that it is not possible to achieve linear phase over the entire Nyquist interval.

FIR response models are preferred over IIR models in practical applications such as signal processing [1, 2], telecommunications [8], and control systems [9-11]. However, the IIR model often arises from signal and system modeling and controller and filter design [1, 2, 11], so it is still used in

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practice. Since the FIR model is more popular than the IIR model, efficient methods are often used to approximate the IIR model using the FIR model.

The IIR model approximation problem can be stated as follows: Given an IIR model $\mathbf{G}(z)$, which is a stable rational transfer function, find $\mathbf{T}(z) = t_0 + t_1 z^{-1} + ... + t_{m-1} z^{-m+1}$ such that the norm $\|\mathbf{F}\|$, which is the transfer function error $\mathbf{F}(z) = \mathbf{G}(z) - \mathbf{T}(z)$, is minimized, choosing which standard to evaluate the error due to the requirements of each problem. Due to the existence of an approximation problem in the design of IIR filters, they are usually of high order. High-order filters will lead to many disadvantages in the process of using filters, so in the filter design process, it is necessary to apply model reduction methods to reduce the order of high-order filters. The problem of model order reduction can be stated as follows: Consider a linear, continuous, time-invariant parameter system with many inputs and many outputs, described in state space by the following system of equations:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \quad (1)$$

in which $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in \mathbf{R}^p$, $\mathbf{y} \in \mathbf{R}^q$, $\mathbf{A} \in \mathbf{R}^{nxn}$, $\mathbf{B} \in \mathbf{R}^{nxp}$, and $\mathbf{C} \in \mathbf{R}^{qxn}$. The goal of the order reduction problem for the model described by the system of (1) is to find the model described by the system of equations:

$$\begin{aligned} \dot{\mathbf{x}}_r &= \mathbf{A}_r\mathbf{x}_r + \mathbf{B}_r\mathbf{u} \\ \mathbf{y}_r &= \mathbf{C}_r\mathbf{x}_r \end{aligned} \quad (2)$$

where $\mathbf{x}_r \in \mathbf{R}^r$, $\mathbf{u} \in \mathbf{R}^p$, $\mathbf{y}_r \in \mathbf{R}^q$, $\mathbf{A}_r \in \mathbf{R}^{rxr}$, $\mathbf{B}_r \in \mathbf{R}^{rxp}$, and $\mathbf{C}_r \in \mathbf{R}^{qxr}$, with $r \le n$ so that the model described by the system of (2) can replace the model described by the system of (1), and at the same time satisfy some of the following requirements:

- The order reduction error is small and can be evaluated.
- The order reduction algorithm needs to calculate efficiently and stability.
- The order reduction algorithm can be performed automatically based on the formula for calculating the upper bound of the order reduction error.
- Important properties of the root system, such as stability and passivity, should be preserved in a reduced order system.
- It must be suitable for each specific requirement of each order reduction problem.

Over the years, hundreds of studies have been published that dealt with solving the problem of order reduction of high-order models. Most of them focused on solving the problem of order reduction for linear systems. Depending on the properties of the original system that need to be preserved in the order reduction system, there are many different order reduction methods. However, they can be classified based on the following basic techniques:

- Singular Perturbations Analysis (SPA) [12].
- Modal Analysis (MA) [13-16].
- Singular Value Decomposition (SVD) analysis [19].
- Moment Matching (MM or Krylov methods) [17].
- Combined SVD and MM [18].

Among the above order reduction techniques, we pay the most attention to the group of methods based on SVD analysis and information from the single Hankel values of the system. The most important proposal of this group of methods is the balanced truncation method [20]. The balanced truncation method is implemented by applying the equivalence condition to the simultaneous diagonalization of the control and observed Gramian matrix of the system. Through the process of diagonalizing two Gramian matrices, it is possible to convert the original model represented in any basis system into an equivalent system representing the coordinate system in internal equilibrium space. From that equilibrium space, the low-order model can be found by removing the eigenvalues that contribute little to the relationship between the input and the output of the system, as well as the states that are less controlled and observed. The balanced truncation method was further developed in [21], and the relationship with Hankel's norm was determined in [22, 23]. In addition to the balanced truncation method, the SVD-based method has a number of other methods such as the stochastic balancing method [24], [25], the LQG balancing truncation algorithm [26], etc. Each order model reduction algorithm has its advantages and disadvantages and should be used for appropriate cases.

In this paper, we are most interested in the LQG balancing truncation algorithm [26] and use it to reduce the high-order filter.

II. THE LQG BALANCED TRUNCATION ALGORITHM

Given a linear, continuous, time-invariant parameter system with many inputs and outputs, described in state space by (1) and according to the balanced truncation algorithm, to determine the transition matrix $\mathbf{T}$, we need to determine the control grammian matrix and the observed gramian matrix by solving a system of Lyapunov equations. However, the condition for the system of Lyapunov equations to have a solution is that the original system (1) must be stable. If the original system (1) is unstable, we cannot solve the system of Lyapunov equations. To solve this problem, the LQG balanced truncation algorithm [26] proposes to calculate the controllable Gramian matrix and the observable Gramian matrix of the unstable system through an extended Riccati system of equations. Once the control and observation Gramian has been determined, it becomes possible to determine the transition matrix $\mathbf{T}$. The detailed content of the LQG balanced truncation algorithm [26] is as follows:

Input: The sytem $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is described in (1) (unstable system).

Step 1: Calculate the control gramian matrix $\mathbf{P}$ and the observed gramian matrix $\mathbf{Q}$ according to the extended Riccati system of equations as follows:

$$\mathbf{AP}+\mathbf{PA}'-\mathbf{PC}'\mathbf{C}'\mathbf{P}+\mathbf{BB}'=0$$
$$\mathbf{A}'\mathbf{Q}+\mathbf{QA}-\mathbf{QBB}'\mathbf{Q}+\mathbf{C}'\mathbf{C}=0$$

Step 2: Determine the triangular matrix on $\mathbf{R}$ by Cholesky analysis of the controlled gramian matrix:

$$\mathbf{P}=\mathbf{RR}^T$$

Step 3. Use matrix $\mathbf{R}$ to calculate SVD analysis of control gramian matrix as follows:

$$\mathbf{RQR}^T=\mathbf{U\Lambda V}^T$$

Step 4: Calculate matrix $\mathbf{L}=\mathbf{V}^{1/2}$.

Step 5: Calculate the non-singular matrix $\mathbf{T}^{-1}=\mathbf{R}^T\mathbf{U}\mathrm{L}^{-1/2}$.

Step 6: Calculate $(\mathbf{A},\ \mathbf{B},\ \mathbf{C})=(\mathbf{T}^{-1}\mathbf{AT},\ \mathbf{T}^{-1}\mathbf{B},\ \mathbf{CT})$.

Step 7: Choose r such that $r < n$, where r is the order of the reduced system. Represent $(\mathbf{A},\ \mathbf{B},\ \mathbf{C})$ in block form as follows:

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{A}_{21} & \mathbf{A}_{22}\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}\mathbf{B}_1\\ \mathbf{B}_2\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}\mathbf{C}_1 & \mathbf{C}_2\end{bmatrix}$$

where $\mathbf{A}_{11}\in R^{rxr}$, $\mathbf{B}_1\in R^{rxp}$, $\mathbf{C}_1\in R^{qxr}$.

Output: Reduced system $(\mathbf{A}_{11},\ \mathbf{B}_1,\ \mathbf{C}_1)$.

III. REDUCING THE ORDER OF THE DIGITAL FILTER

Consider a IIR digital filter with a transfer function in the form of $\mathbf{H}(z)$ in (3) and the general structure of a laddered IIR filter [1-2]:

$$\mathbf{H}(z)=\frac{b_0z^N+b_1z^{N-1}+...+b_{N-1}z+b_N}{a_0z^N+a_1z^{N-1}+...+a_{N-1}z+a_N} \quad (3)$$

In [27], the authors have determined the transfer function model of the 6th-order IIR digital filter as follows:

$$\mathbf{H}(z)=\frac{\mathbf{A}(z)}{\mathbf{B}(z)}$$

with:

$$\mathbf{A}(z)=-0.1242z^5+0.1581z^4+0.5273z^3+0.2154z^2-0.0647z+0.6889$$
$$\mathbf{B}(z)=z^6-1.095z^5+1.299z^4-1.113z^3+1.028z^2-0.6043z+0.426$$

The 6th-order IIR digital filter has a non-minimum phase form and has a polarity very close to the unit circle in the z-plane. Therefore, it is prone to digital errors and is not suitable for performing Digital Signal Processing (DSP). At the same time, the order of the digital filter is high, so when used in practice, there will be many disadvantages. To overcome this implementation difficulty, we need to reduce the order of the IRR digital filter. To do so, we convert the 6th-order digital filter to the form of linear analog filter through the transformation $z=s+1$. The obtained result is:

$$\mathbf{H}(s)=\frac{\mathbf{C}(s)}{\mathbf{D}(s)}$$

with:

$$\mathbf{C}(s)=-0.1242s^5-0.4629s^4-0.0823s^3+1.504s^2+1.959s+1.401$$
$$\mathbf{D}(s)=s^6+4.905s^5+10.82s^4+13.13s^3+9.533s^2+3.834s+0.9407$$

Performing order reduction of the 6th-order digital filter according to the steps of the LQG balanced truncation algorithm [26], the results are:

Step 1: The control Gramian matrix $\mathbf{P}$ and the observed Gramian matrix $\mathbf{Q}$ have the following form:

$$\mathbf{P}=\begin{bmatrix}0.1607 & 0.0009 & -0.2016 & -0.0036 & 0.0943 & -0.0013\\ 0.0009 & 0.4125 & 0.0118 & -0.4250 & -0.0607 & 0.0813\\ -0.2016 & 0.0118 & 0.8439 & 0.0004 & -0.8583 & -0.0320\\ -0.0036 & -0.4250 & 0.0004 & 0.8738 & 0.1547 & -0.3196\\ 0.0943 & -0.0607 & -0.8583 & 0.1547 & 1.8095 & 0.1140\\ -0.0013 & 0.0813 & -0.0320 & -0.3196 & 0.1140 & 0.3995\end{bmatrix}$$

$$\mathbf{Q}=\begin{bmatrix}0.3205 & 0.4039 & 0.4465 & 0.5228 & 0.3373 & 0.3733\\ 0.4039 & 0.5175 & 0.5839 & 0.6999 & 0.4634 & 0.5471\\ 0.4465 & 0.5839 & 0.6753 & 0.8327 & 0.5671 & 0.7160\\ 0.5228 & 0.6999 & 0.8327 & 1.0586 & 0.7419 & 0.9968\\ 0.3373 & 0.4634 & 0.5671 & 0.7419 & 0.5336 & 0.7543\\ 0.3733 & 0.5471 & 0.7160 & 0.9968 & 0.7543 & 1.1679\end{bmatrix}$$

Step 2-4: The matrices $\mathbf{R},\mathbf{U}$, and $\mathbf{L}$ have the following form:

$$\mathbf{R}=\begin{bmatrix}0.4008 & 0.0023 & -0.5030 & -0.0089 & 0.2353 & -0.0033\\ 0 & 0.6423 & 0.0202 & -0.6616 & -0.0954 & 0.1266\\ 0 & 0 & 0.7685 & 0.0121 & -0.9604 & -0.0471\\ 0 & 0 & 0 & 0.6602 & 0.1596 & -0.3563\\ 0 & 0 & 0 & 0 & 0.8929 & 0.1551\\ 0 & 0 & 0 & 0 & 0 & 0.4798\end{bmatrix}$$

$$\mathbf{U}=\begin{bmatrix}-0.0412 & 0.0204 & -0.5517 & -0.7850 & 0.1946 & -0.1985\\ -0.1800 & -0.1740 & -0.2263 & -0.1283 & -0.8878 & 0.2855\\ -0.1460 & -0.6028 & -0.4883 & 0.4562 & 0.0765 & -0.4037\\ 0.3772 & -0.7024 & 0.1683 & -0.2285 & 0.2089 & 0.4900\\ 0.7612 & -0.0154 & 0.1140 & -0.0702 & -0.3421 & -0.5342\\ 0.4720 & 0.3352 & -0.6038 & 0.3195 & 0.0868 & 0.43669\end{bmatrix}$$

$$\mathbf{L}=\begin{bmatrix}1.0691 & 0 & 0 & 0 & 0 & 0\\ 0 & 0.355 & 0 & 0 & 0 & 0\\ 0 & 0 & 0.0109 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.0078 & 0 & 0\\ 0 & 0 & 0 & 0 & 0.0065 & 0\\ 0 & 0 & 0 & 0 & 0 & 0.0001\end{bmatrix}$$

Step 5: The non-singular matrix $\mathbf{T}$ has the form:

$$\mathbf{T}=\begin{bmatrix} 0.4244 & 0.5900 & 0.7316 & 0.9695 & 0.7049 & 1.0172 \\ -0.5988 & -0.6398 & -0.5389 & -0.3893 & -0.0826 & 0.4162 \\ -0.2020 & -0.0591 & -0.0283 & -0.0529 & 0.0361 & -0.1311 \\ -0.1191 & -0.0274 & 0.0346 & 0.0053 & -0.0172 & 0.0589 \\ 0.0182 & -0.0757 & -0.0336 & 0.0415 & -0.0335 & 0.0146 \\ -0.0203 & 0.0185 & -0.0160 & 0.0158 & -0.0084 & 0.0101 \end{bmatrix}$$

Step 6: The system is in equilibrium equivalent form:

$$\mathbf{A}=\begin{bmatrix} 0.0120 & -0.4910 & 0.0785 & 0.0465 & 0.0073 & -0.0081 \\ 0.4910 & -0.4415 & 0.3529 & 0.2060 & 0.0301 & -0.0343 \\ 0.0785 & -0.3529 & -1.8800 & -1.2871 & -0.8508 & 0.3832 \\ 0.0465 & -0.2060 & -1.2871 & -0.9049 & -1.6577 & 0.3143 \\ -0.0073 & 0.0301 & 0.8508 & 1.6577 & -0.0255 & 0.0558 \\ 0.0081 & -0.0343 & -0.3832 & -0.3143 & 0.0558 & -1.6653 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} 0.4244 \\ -0.5988 \\ -0.2020 \\ -0.1191 \\ 0.0182 \\ -0.0203 \end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} 0.4244 & 0.5988 & -0.2020 & -0.1191 & -0.0182 & 0.0203 \end{bmatrix}$$

The results of the order reduction of the digital filter are shown in Table I.

TABLE I. RESULT OF ORDER REDUCTION OF THE 6TH ORDER FILTER

Order	$\mathbf{H_r(s)}$	$\|\mathbf{H}-\mathbf{H}_r\|_\infty$
5	$\frac{-0.1238s^4-0.2611s^3+0.405s^2+0.7221s+0.8147}{s^5+3.24s^4+5.188s^3+4.469s^2+1.952s+0.5468}$	$9.7456.10^{-4}$
4	$\frac{-0.1235s^3-0.2711s^2+1.003s+0.01749}{s^4+3.214s^3+1.635s^2+0.7117s+0.01141}$	0.0437
3	$\frac{-0.1376s^2-0.08334s+0.6907}{s^3+2.309s^2+1.161s+0.4662}$	0.0266
2	$\frac{-0.1784s+0.3335}{s^2+0.4294s+0.2358}$	0.0904

The step response and the bode response are two basic responses that evaluate the quality of the filter. Therefore, we will use them to evaluate the quality of the low-order digital filter. The step response and bode response of the low-order digital filters are shown in Figures 1-4. We can see that the step response of the 5th-order digital filter coincides with the step response of the 6th- order digital filter (Figure 1). In Figure 2 we see that the step response of the 3rd-order digital filter almost coincides with the step response of the 6th-order digital filter and the step response of the 4th-order digital filter has a small deviation from the step response of the 6th-order digital filter. The step response of the 2nd-order digital filter has a large deviation from the step response of the 6th-order digital filter. Figure 4 shows the bode response of the 5th-order digital filter. It is noted that the 4th-order digital filter completely coincides with the bode response of the 6th-order digital filter.

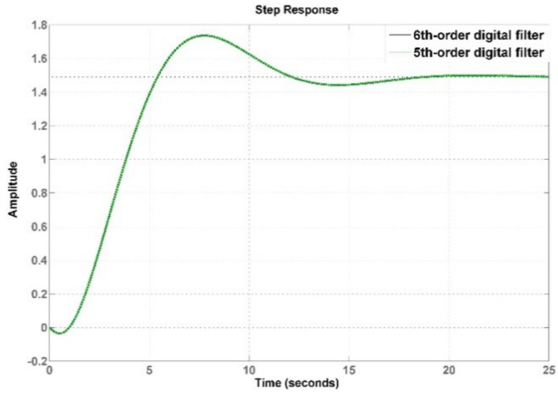

Fig. 1. Step response of 5th- and 6th-order digital filters.

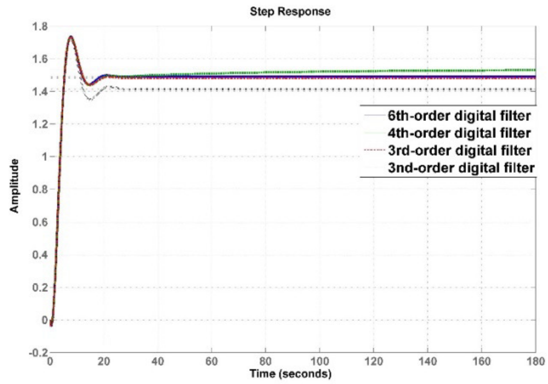

Fig. 2. Step response of 6th-, 4th-, 3rd-, and 2nd-order digital filters.

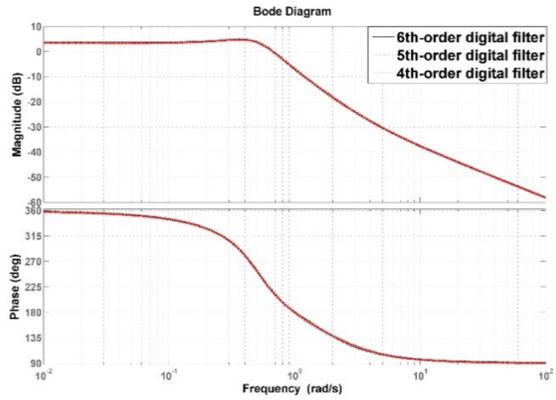

Fig. 3. Bode response of the 6th-, 5th-, and 4th-order digital filters.

In Figure 4, we see that:

- In the frequency range ω < 9.37rad/s, the frequency amplitude response of the 3rd-order digital filter coincides with the frequency amplitude response of the 6th-order digital filter.
- In the frequency range ω > 9.37rad/s, the frequency amplitude response of the 3rd-order digital filter has a small deviation from the frequency amplitude response of the 6th-order digital filter.

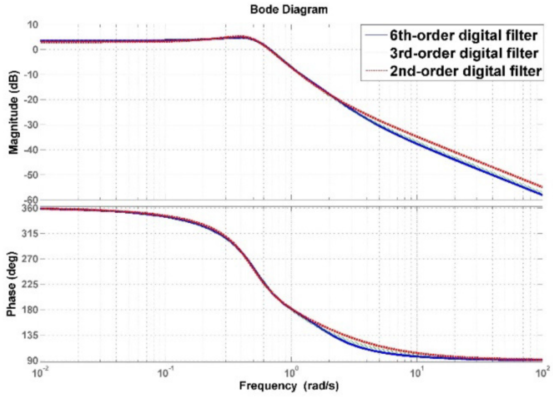

Fig. 4. Bode response of the 6th-, 3rd-, and 2nd-order digital filters.

- In the frequency range $\omega < 2.07$rad/s and $\omega > 12.5$rad/s, the frequency phase response of the 3rd-order digital filter coincides with the frequency phase response of the 6th-order digital filter.
- In the frequency range 2.07rad/s < ω < 12.5rad/s, the frequency phase response of the 3rd-order digital filter has a small deviation from the frequency phase response of the 6th-order digital filter.
- In the frequency range $\omega < 2.7$rad/s, the frequency amplitude response of the 2nd-order digital filter coincides with the frequency amplitude response of the 6th-order digital filter.
- In the frequency range $\omega > 2.7$rad/s, the frequency response of the 2nd-order digital filter has a large deviation from the frequency response of the 6th-order digital filter.
- In the frequency range $\omega < 1.33$rad/s and $\omega > 29.8$rad/s, the frequency phase response of the 2nd-order digital filter coincides with the frequency phase response of the 6th-order digital filter.
- In the frequency range of 1.33rad/s < ω < 29.8rad/s, the frequency phase response of the 2nd-order digital filter has a large deviation from the frequency phase response of the 6th-order digital filter.

Comment: If minimum order reduction error is desired, the 5th-order digital filter can be used instead of the 6th-order digital filter. If we want a low-order digital filter with the smallest order, but with quality almost equivalent to that of a 6th-order digital filter, we can choose a 3rd-order digital filter to replace the 6th order digital filter.

The novelty of the current paper is that the LQG algorithm has been used to determine a low-order filter that can replace the high-order filter. The paper has evaluated the error of order reduction according to both the formula for evaluating the order reduction error and the error on step response characteristics and bode response characteristics. In [27], the author does not specify the order of the order reduction system, only focuses on converting the filter from IIR form to FIR form by order reduction algorithms and then evaluates the limit of order reduction error at some frequencies. In the future, we will evaluate the order reduction error at frequency points to compare the obtained results with the results in [27].

IV. CONCLUSION

Digital filters have many advantages, so they are widely used in technical fields. The most common digital filter design method is usually to design first an analog filter and then convert it to digital filter form. Digital IRR filters often arise in engineering problems, so they are increasingly utilized and studied in research. The design of digital IRR filters often has to apply approximation methods, so the resulting filter is often of high order. To make the digital IRR filter simple and low in computational cost, we need to apply model order reduction algorithms to reduce the order of the high-order IRR filter. This article has applied the LQG balanced truncation algorithm to reduce the order of a 6th-order digital filter. The results of comparison and evaluation of low-order filters show that the resulting 5th-order digital filter can replace the 6th-order digital filter without any change in filter quality. Also, a 3rd-order digital filter can be used instead of a 6th-order digital filter if it is accepted that the quality of the low-order filter is only approximately that of the 6th-order filter. The simulation results show the correctness of the filter order reduction results using the LQG balanced truncation algorithm.

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