

# A New Computational Envelope Solution for Helical Gear Disc Tool Profiling

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## ABSTRACT

This paper presents a new computational envelope solution for profiling the helical gear disc tool. It uses the normal projection of the disc tool axis onto the helical gear surface to generate the characteristic curve and then automatically computes the geometric data of the characteristic curve to create the disc tool in 3D solid form. As a popular profile, the ISO heliacal gear was a typical proper example to verify and clarify the proposed solution. The solution can quickly create the helical gear disc tool in 3D solid form with high accuracy. The 3D comparison average error of the helical gear disc tool surfaces generated by the proposed solution and the Boolean method is 0.004 mm, and the RMS error is 0.009 mm.

*Keywords-disk tool; gear grinding; characteristic curve; singular point; envelope method; computer aided design*

## I. INTRODUCTION

Typical gear machining operations are milling and grinding with disc tools. Conventional disc tool profiling is deduced from the envelope theory, which determines the envelope surfaces of surface families. Authors in [1] published general theoretical studies on determining the envelope to a family of surfaces by giving the meshing equations of kinematics pairs, including pinion and gear as well as cutting tool and gear. More recently, numerical solutions and algorithms for the envelope problem have been suggested. Authors in [3] adopted the disc tool envelope theory to gain the uniform wear of the disc tool when grinding screw compressor rotors. Authors in [4] quantified the disc tool setting errors, such as the axis angle mismatch and position mismatch, to compensate for the disc tool deformation and wear. Authors in [7-9] presented 3D CAD-based envelope methods putting into service the Boolean operator to design disc tools machining helical surfaces. Authors in [10-13] proposed an envelope method established on the CATIA graphical platform for profiling rotation tools, generating constant pitch cylindrical helical surfaces. Each method has specific advantages and disadvantages and serves a particular purpose. The purely analytical methods constitute the fundament of the rest approaches and offer precise solutions. Still, they are complex and more difficult in the case of undercutting or singular points. Besides, the meshing equations can only be solved accurately in some exceptional cases and require a surface representation through a system of equations. The numerically analytical methods come only from the purely analytical methods, which solve approximately a system of five

nonlinear meshing equations, so they have some drawbacks similar to those of the original analysis method. The Boolean method stems from a novel idea, which simulates the actual envelope process like the material cutting process, so the surface is not required to be regular and without any singular points. Still, the former method takes a long time to perform and cannot be directly utilized for reverse engineering because the object is not solid. The solutions engaging the normal projection command in the CATIA graphical platform are exciting but cannot be performed in some cases.

Based on the above mentioned works, this study aims to create a new computational solution for calculating the helical gear disc tool profiles: Given the helical gear surface in 3D solid form, the present study attempts to create its disk tool in 3D solid form with its proper position.

## II. A NEW PROPOSED ENVELOPE COMPUTATIONAL SOLUTION FOR GEARS DISC TOOL PROFILING

The meshing condition of a kinematic pair, including pinion and gear pair and gear and cutting tool pair, is derived from the envelope concept to a family of surfaces. Authors in [1] generally denote  $\Sigma_1$  and  $\Sigma_2$  as a meshing surface pair, including pinion and gear or machined gear surface and its envelope cutting tool. The coordinate systems  $S_1$  and  $S_2$  are fixed into  $\Sigma_1$  and  $\Sigma_2$ .  $\Sigma_1$  is presented by a vector function  $r_1(u, \theta)$ ,  $r_2(u, \theta, \tau)$  is the vector function presenting the surfaces family  $\Sigma_1$  represented in  $S_2$ , where  $\tau$  is the motion parameter. As a

meshing condition,  $\Sigma_1$  and  $\Sigma_2$  are tangent together, which deduces (1) [1]:

$$\left( \frac{\partial r_2}{\partial u} \times \frac{\partial r_2}{\partial \theta} \right) \cdot \frac{\partial r_2}{\partial \tau} = 0 \tag{1}$$

where  $\partial r_2/\partial u$  and  $\partial r_2/\partial \theta$  represent two tangent lines of the common tangent plane,  $\partial r_2/\partial \tau$  represents the relative velocity at a contact point, which leads to another form of the meshing condition [1]:

$$N.V = 0 \tag{2}$$

where  $N = (\partial r_2/\partial u \times \partial r_2/\partial \theta)$  represents a normal vector to surface  $r_1$  and  $V = \partial r_2/\partial \tau$  represents a relative velocity vector of the points on the surfaces  $r_1$  with respect to the surfaces  $r_2$ .

In the case of the helical gear surface and its disc tool surface, (2) deduces (3), used for calculating the characteristic curve:

$$(\vec{A}, \vec{N}_\Sigma, \vec{r}_1) = 0 \tag{3}$$

where  $\vec{A}$  presents the disc tool axis,  $\vec{N}_\Sigma$  presents a normal vector of the machined gear surface at the contact point, and  $\vec{r}_1$  presents the position vector of the contact point. These contact points belong to the characteristic curve.

In the CATIA platform, such characteristic curves can be created by projecting the disc tool axis normal to the helical gear surface, as shown in Figure 1.

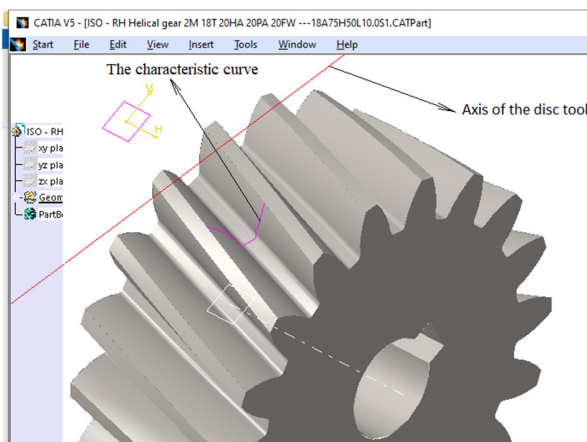


Fig. 1. The characteristic curve was created in the CATIA platform.

The heliacal gear 3D model with the characteristic curve is imported into the AutoCAD platform and a subroutine written in Autolisp is run following the algorithm portrayed in Figure 2. The sentences in the algorithm blocks are written similarly to the commands in Autolisp language. Some explanations are:

- The sentence "Input the characteristic curve by a selection set of geometry entities" is performed by the command "ssget".
- The statement "k:= the number of entities in the selection set" is performed by the commands "(setq k (sslenght taphop))".

- The statement "ent-list:= Returns an association list containing the entity definition of ent-name" is performed by the commands "(setq ds (entget tenmoi))".

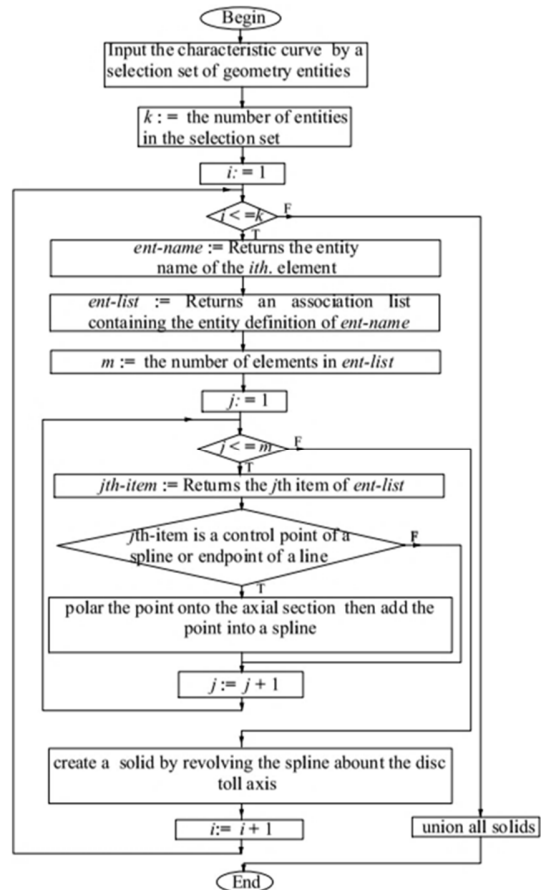


Fig. 2. The algorithm for the autolisp subroutine that creates the disc tool 3D model.

Some other statements can also be found in the Autolisp code of the subroutine below.

```
(defun C:disctool ()
  (setq taphop (ssget))
  (setq k (sslenght taphop))
  (setq entno 0)
  (repeat k
    (setq tenmoi (ssname taphop entno))
    (setq ds (entget tenmoi))
    (setq lap (length ds))
    (setq lap (- lap 1))
    (setq n 1)
    (setq tam (list 0 0 0))
    (command "_3dpoly")
    (setq flag 0)
    (repeat lap
      (progn
        (setq diem3d (cdr (nth n ds)))
        (setq diem3d (cdr (nth n ds))))))
    (setq entno (+ entno 1))
  )
)
```

```

    (setq goc0 (angle tam
diem3d))
    (setq x (nth 0 diem3d))
    (setq y (nth 1 diem3d))
    (setq z (nth 2 diem3d))
    (setq tamdong (list 0.000
0.000 z))
    (setq d (distance diem3d
tamdong))
    (setq diemmoi (polar tamdong
0.000 d))
    (setq p2 (polar tamdong
0.000 5))
    (Command diemmoi)
    (print n)
    (if (= flag 0)
      (progn
        (setq diemdau diemmoi)
        (setq p1 (polar tamdong
0.000 5))
          (setq flag 1)
        )
      )
    )
    (setq n (+ n 1))
  )
  (setq segment (entlast))
  (setq diemcuoi diemmoi)
  (command "line" p1 diemdau "")
  (setq e1 (entlast))
  (command "line" p2 diemmoi "")
  (setq e2 (entlast))
  (command "line" p1 p2 "")
  (setq e3 (entlast))
  (command "region" e1 e2 e3 segment "")
  (setq p4 (list 0.00 0.00 10))
  (setq p5 (list 0.00 0.00 50))
  (command "revolve" (entlast) "" p4 p5
360)
  (ssadd (entlast) ss)
  (setq entno (+ entno 1))
)
(command "union" ss "")
)

```

Based on the algorithm illustrated in Figure 2, in the AutoCAD package, the Autolisp subroutine above picks up every point on the characteristic curves to calculate the coordinates of the points on the disc tool axial section to create the disc tool 3D model.

### III. TESTING RESULTS AND DISCUSSION

As a testing sample, the ISO helical gear was created by the SolidWork Package [14, 15] with the main parameters set: Module = 2, the number of teeth = 18, pressure angle = 20, helix angle = 20, as depicted in Figure 3.

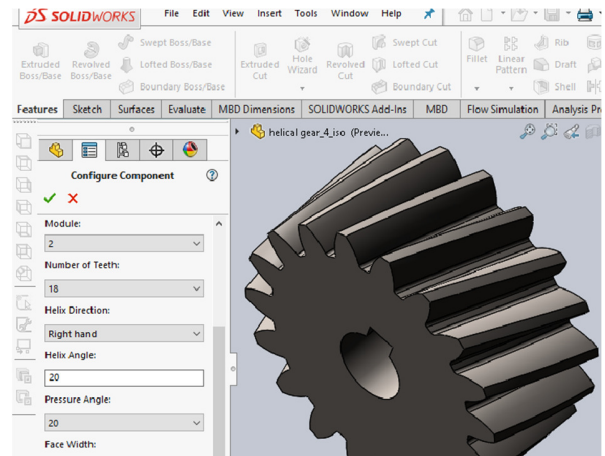


Fig. 3. The helical gear was created in SolidWorks.

The helical gear was imported into the CATIA package and the disc tool axis was created with the following position parameters: the axis distance is 80 mm, the axis angle is  $73^{\circ}$ , the distance from the coordinate origin to the disc tool axis is 9 mm, as evidenced in Figure 4. The project command in the CADTIA package was used and the characteristic curve was created (see again Figure 1). The gear, the characteristic curve, and the disc tool axis were imported into the AutoCAD package, then the subroutine was run, according to the algorithm shown in Figure 2, written in the Autolisp to create the disc tool, as displayed in Figure 5.

By deploying the Geomagic Control X Package, the disc tool constructed by the proposed solution was compared with the disc tool constructed by the Boolean method [7-9], as exhibited in Figures 6, 7, and Table I.

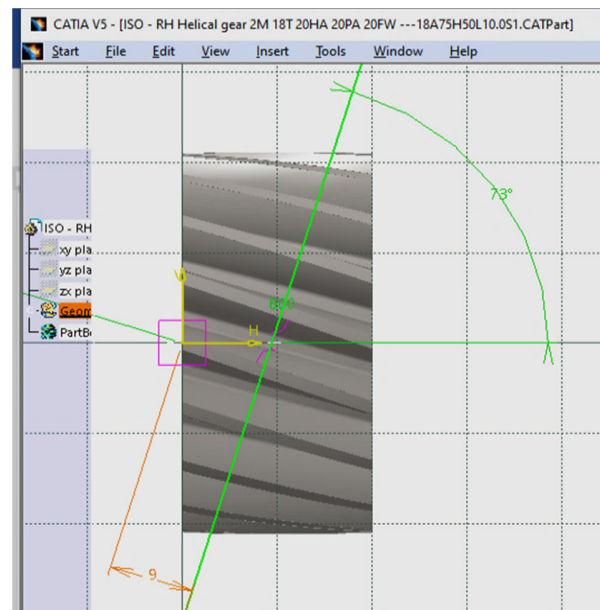


Fig. 4. Disc tool position parameters.

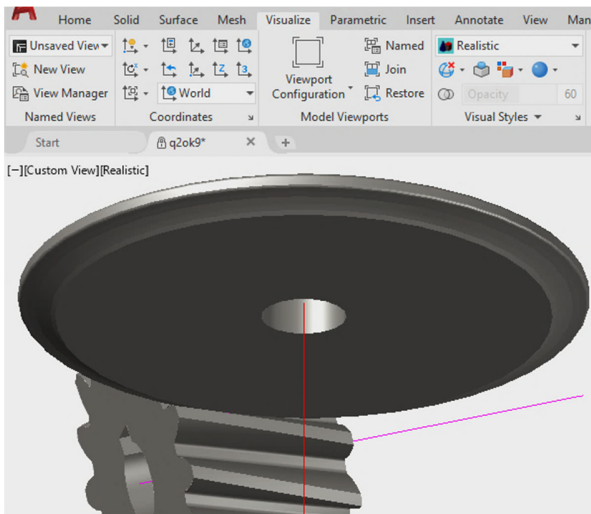


Fig. 5. The AutoLISP subroutine creates the disc tool 3D model.

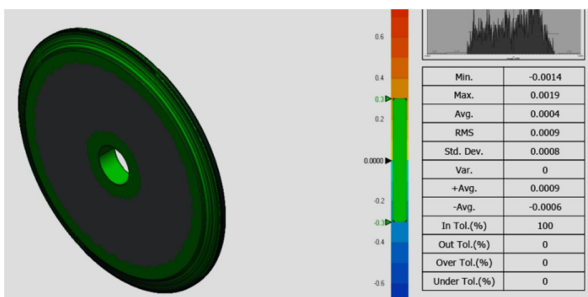


Fig. 6. 3D surface comparison of two disc tools generated by the two methods.

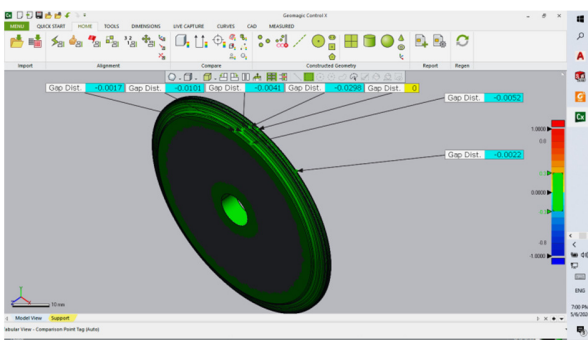


Fig. 7. 3D point comparison of the two generated disc tools.

TABLE I. 3D POINT COMPARISON OF TWO DISC TOOLS GENERATED BY THE TWO METHODS

Name	Reference Position			Measured Position			Gap Dist.
	X	Y	Z	X	Y	Z	
CMP1: 1	0	29.7692	54.1687	0	29.7683	54.1672	-0.0017
CMP1: 2	2	31.2588	53.3827	1.9999	31.2574	53.3789	-0.0041
CMP1: 3	4	32.9342	52.7757	3.9975	32.9142	52.7536	-0.0298
CMP1: 4	6	32.8044	51.8436	6	32.8044	51.8436	0
CMP1: 5	2	30.406	53.7534	1.9997	30.4019	53.7442	-0.0101
CMP1: 6	19.9999	26.3924	51.1594	19.9992	26.3915	51.1613	-0.0022
CMP1: 7	5.9999	29.1582	54.1685	5.9994	29.1556	54.1641	-0.0052
Min.	0.0000	26.3924	51.1594	0.0000	26.3915	51.1613	-0.0298
Max.	19.9999	32.9342	54.1687	19.9992	32.9142	54.1672	0.0000

The deviations depend not only on the accuracy of the proposed solution, but also on the accuracy of the Boolean method [7-9, 15], which, naturally, is not absolutely accurate, while the proposed method comes from the traditional envelope theory, so it is absolutely accurate. In addition, the Boolean method takes a long time to be implemented.

The proposed solution has expanded the solution provided in [10-13], by adding an algorithm and an Autolisp subroutine. It automatically computes the geometric data of the characteristic curve to calculate the coordinates of the points on the axial disc tool section and then creates the disc tool 3D model. In contrast, the CADTIA-based solution [10-15] can only create a surface by employing the revolving surface command, which cannot be completed in some cases.

IV. CONCLUSION

This paper presents a new envelope computational solution that comes from the traditional envelope theory for profiling the helical gear disc tool. It uses the normal projection of the disc tool axis onto the helical gear surface to generate the characteristic curve. After that it automatically computes the geometric data of this characteristic curve to calculate the coordinates of the points on the axial disc tool section and then creates the disc tool 3D model. The experimental validation result in the ISO heliacal gear case leads to the conclusion below.

The proposed solution does not require helical gear surface representation through equations. The solution is not difficult to be implemented and can quickly generate the helical gear disc tool 3D model with high accuracy. The 3D comparison average error of the helical gear disc tool surface emerging by the proposed method and the Boolean method is 0.004 mm, and their RMS error is 0.009 mm. The proposed solution is universal, so it could be effectively deployed for the meshing gear pairs as well as for other kinematic pairs, such as the hob and gear, which will be addressed in future research.

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