Machine Learning-enhanced Direction-of-Arrival Estimation for Coherent and Non-Coherent Sources

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ABSTRACT

Accurate Direction-Of-Arrival (DOA) estimation for both coherent and non-coherent sources remains a critical challenge in array signal processing, particularly under sparse sensor configurations. This study introduces a novel 3D coprime array method that enhances source separation and spatial resolution. By leveraging a unique joint diagonalization framework with a full-rank Toeplitz matrix, the proposed approach effectively decorrelates coherent sources while preserving the accuracy of uncorrelated signals. A machine learning model can be employed to further refine the DOA estimates, utilizing a regression

model or neural network to predict DOA based on features extracted from the covariance matrix. A new cost function, independent of the number of sources, is proposed to increase robustness in complex environments. Extensive simulations demonstrate that the proposed technique significantly outperforms established algorithms, including 3D Unitary Root-MUSIC, modified Root-MUSIC, ECA-MURE, and FBSS. The results reveal substantial improvements in Root Mean Square Error (RMSE) across various Signal-to-Noise Ratios (SNRs), affirming the method's effectiveness. Additionally, the approach's adaptability to different scenarios makes it suitable for real-world applications. These advances pave the way for improved applications in Unmanned Aerial Vehicles (UAVs), radar systems, and next-generation communication networks.

Keywords-DOA estimation; 3D coprime array; coherent sources; Toeplitz matrix; joint diagonalization; signal processing; spatial spectrum; direction finding

I. INTRODUCTION

The estimation of Direction-of-Arrival (DOA) for coherent signals is crucial in various applications such as radar, wireless communications, and sensor networks. However, precise DOA estimation remains a significant challenge, particularly when dealing with coherent sources and a limited array of data. The difficulty arises due to the correlation between signals, which degrades the performance of conventional subspace-based methods such as MUSIC and ESPRIT. In recent years, various approaches have been proposed to address this challenge by leveraging advanced array structures, signal processing techniques, and hardware optimizations.

This study presents an extension of a previously proposed approach that boosts the estimate of coherent sources and improves spatial resolution by introducing a 3D coprime array configuration. This extends previous work on signal decorrelation techniques and sparse array processing, which showed notable performance gains when used for DOA estimation. In the case of 3D sparse arrays, the application of the Nyström approximation has been investigated in the Unitary Root-MUSIC technique, achieving significant improvements in estimation accuracy while retaining processing efficiency [1, 2]. Furthermore, the core of the strategy suggested in [3] was a robust DOA estimation in coprime arrays, and the adaptive spectral analysis techniques in later studies provide a promising foundation for this.

The fundamental feature of this method is the decorrelation of coherent sources using a full-rank Toeplitz matrix, which enables a more precise estimation of DOA without requiring prior knowledge of the number of sources. The ECA-MURE algorithm is further enhanced by utilizing coprime arrays for high-precision DOA estimation [4]. The significance of optimizing sensor arrays for coherent and uncorrelated source estimation is further highlighted by recent developments in sensor array processing, such as the joint diagonalization structures and manifold reconstruction methods [5, 6]. The TMS320C6678 Digital Signal Processor (DSP), a powerful platform renowned for its effective vector instruction execution, was used to execute the suggested algorithm. For real-time DOA estimation, previous research has shown that it is feasible to apply sophisticated signal processing techniques on DSP platforms [7]. The proposed method increases computing performance by performing numerous single-precision floating-point operations simultaneously, taking advantage of the processor's 128-bit vector execution capabilities. Recent efforts on DSP-based DOA estimation for coherent sources [8] showed that this is especially advantageous for real-time applications.

This study integrates a 3D coprime array with Toeplitz matrix diagonalization and an effective DSP-based implementation to propose a novel enhancement to current DOA estimation methods. By addressing coherent source separation, a persistent issue, this method provides a solid solution that enhances computational efficiency and spatial resolution. Comprehensive simulations and real-world experiments showed that the suggested method performs better in real-time and accuracy than existing methods, making it appropriate for a variety of applications such as UAVs, radar, and next-generation communication systems.

II. SIGNAL MODEL FOR ENHANCED DOA ESTIMATION FOR COHERENT SOURCES USING 3D COPRIME ARRAYS AND TOEPLITZ MATRIX

This section details the mathematical model underlying the proposed method for DOA estimation of coherent sources using a 3D coprime array and a Toeplitz matrix formulation. The model combines the coprime array configuration with a full-rank Toeplitz matrix-based approach to address the challenges of coherent sources and utilizes the array data to compute accurate DOA estimates. Utilizing the benefits of 3D coprime arrays and Toeplitz matrices, methods can be combined to estimate the DOA of coherent and non-coherent sources simultaneously, without prior knowledge of the number of sources. In a 3D coprime array configuration, let the received signal at the n-th sensor element for the k-th sample be expressed as [9-11]:

$$\mathrm{X}_n(k) = \sum_{l=1}^{L} \mathrm{A}_l\,(k) e^{j\varphi_{l,n}(k)} + n_n(k) \qquad (1)$$

where $\mathrm{A}_l(k)$ is the amplitude of the l-th coherent source, $\phi_{l,n}(k)$ represents the phase shift at the n-th sensor due to the l-th source, and $n_n(k) \sim C_N(0, \sigma^2)$ is the additive white Gaussian noise. For a 3D coprime array, the coordinates of the sensors are defined as:

$$\mathrm{P}_n = (x_n, y_n, z_n) = \left(nd_x, md_y, pd_z\right) \qquad (2)$$

The inter-element spacing's d_x, d_y, and d_z should be coprime to minimize spatial aliasing and enhance angular resolution. The observation vector can be constructed as:

$$\mathrm{Y}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_N(k) \end{bmatrix} = \sum_{l=1}^{L} \mathrm{A}_l\,(k) e^{j\varphi_{l,n}(k)} + \mathrm{N}(k) \qquad (3)$$

where $A_l(k)$ is the array manifold vector defined as:

$$A_l(k) = \begin{bmatrix} e^{j\varphi_{l,1}(k)} \\ e^{j\varphi_{l,2}(k)} \\ \vdots \\ e^{j\varphi_{l,N}(k)} \end{bmatrix} \quad (4)$$

The output covariance matrix $R_X(k)$ of the observation vector can be formulated as:

$$R_X(k) = E\{Y(k)Y^H(k)\} = \sum_{l=1}^{L} E\{A_l(k)A_l^H(k)\} + \sigma^2 I \quad (5)$$

Using the structure of the 3D coprime array, the elements of $R_X(k)$ can be expressed as:

$$R_{i,j}(k) = \sum_{l=1}^{L} A_l(k)A_l^*(k)e^{j(\varphi_{l,i}(k)\ (\varphi_{l,j}(k))} + \sigma^2\delta_{ij} \quad (6)$$

Let us construct a Toeplitz matrix based on the covariance matrix, which captures temporal correlations:

$$T = \begin{bmatrix} R_{xx}(0) & R_{xx}(1) & \dots & R_{xx}(N-1) \\ R_{xx}(1) & R_{xx}(2) & \dots & R_{xx}(N) \\ \vdots & \vdots & \ddots & \vdots \\ R_{xx}(N-1) & R_{xx}(N) & \dots & R_{xx}(2N-2) \end{bmatrix} \quad (7)$$

This matrix utilizes the full spatial-temporal information for improved spectral estimation. Incorporating joint diagonalization techniques allows for simultaneous estimation of DOAs, thus enhancing efficiency:

$$J(\theta) = \min_{\theta}\{tr(R - R_T)^H(R - R_T)\} \quad (8)$$

where R_T is the theoretical covariance matrix constructed from the steering vectors. Let us integrate a machine-learning model for further refinement of the DOA estimates. Use a regression model or neural network to predict the DOA based on features extracted from the covariance matrix:

$$D_{ML} = f(R_X, T) \quad (9)$$

This model can learn from historical data to improve accuracy and robustness. Let us express the pseudo-power spectrum as:

$$P(\theta) = \max_{\theta}\{\lambda(H(\theta)V(\theta))\} \quad (10)$$

where λ represents the eigenvalues, and $H(\theta)$ is constructed from the steering vectors of the 3D coprime array. This innovative model not only leverages the unique properties of 3D coprime arrays but also incorporates advanced signal processing techniques and machine learning methods for enhanced performance in DOA estimation. By addressing the challenges of coherent and non-coherent signals in spatially diverse environments, this approach offers a promising framework for future research and practical applications in various fields, including communications, robotics, and surveillance.

III. RESULTS AND DISCUSSION

Comprehensive simulations were conducted to evaluate the performance of the proposed algorithm. The Root Mean Square Error (RMSE) was calculated as follows [12, 13]:

$$RMSE(\theta) = \sqrt{\frac{1}{T}\sum_{i=0}^{T}\left(\hat{\phi}_i - \phi\right)^2} \quad (11)$$

where $\hat{\phi}$ is the estimated DOA in the t-th trial, and ϕ is the true DOA. The number of sources K, Signal-to-Noise Ratio (SNR), and the number of snapshots (L) varied across experiments to evaluate performance under diverse conditions. Extensive numerical experiments were conducted to validate the effectiveness of the proposed approach. These experiments used Monte Carlo trials (T), where the actual DOA was compared with the estimated one to calculate the RMSE.

To construct the coprime array, comprising $M + N - 1 = 6$, physical sensors located at positions $\{-8d, -6d, -4d, -3d, -2d, 0\}$ with $d = \lambda/2$, the coprime pair of positive integers was chosen as $M = 2$ and $N = 5$. For this simulation, it is assumed that there are four coherent sources $\theta_1 = -10°$, $\theta_2 = 0°$, $\theta_3 = 10°$, and $\theta_4 = 20°$. Additionally, four uncorrelated sources are considered, with DOAs at angles $\theta_1 = -30°$, $\theta_2 = 15°$, $\theta_3 = 45°$, and $\theta_4 = 60°$. The RMSE was computed as a function of the SNR, which ranged from −20 dB to 20 dB. This evaluation was based on 500 Monte Carlo trials and utilized 100 snapshots per trial.

To demonstrate the advantages of the method, its performance was compared with several established algorithms, including 3D-Unitary Root-MUSIC, the modified Root-MUSIC algorithm, ECA-MURE, and FBSS. The proposed approach stands out because it integrates a novel 3D joint diagonalization framework and a full-rank Toeplitz matrix, which enhance source separation and improve spatial resolution.

The comparative performance of this method with 3D-Unitary Root-MUSIC, Modified Root-MUSIC, ECA-MURE, and FBSS was evaluated by replicating their original procedures under a consistent experimental setup. Each algorithm was implemented using the structured 3D sensor array with $\lambda/2$ spacing, ensuring identical signal parameters, SNR values (–20 dB to 20 dB), and 500 Monte Carlo trials. This rigorous framework enabled fair comparisons, validating the RMSE and spatial resolution of the proposed approach while ensuring reproducibility.

The experimental setup involved arranging sensors in a structured 3D configuration along the x, y, and z axes, with 10, 8, and 6 sensors, respectively. The sensors were evenly spaced, with inter-element distances of half a wavelength ($\lambda/2$) along each axis. This arrangement, shown in Figure 1, was carefully designed to optimize the detection of directional signals.

By using a 3D coprime array structure and innovative processing techniques, the proposed method demonstrates significantly better performance compared to existing algorithms. The proposed approach is particularly effective in challenging scenarios involving mixed coherent and uncorrelated sources, making it a robust and reliable solution for accurate DOA estimation. RMSE was measured against the SNR, which varied from -20 dB to 20 dB, using 500 Monte Carlo trials and 100 snapshots.

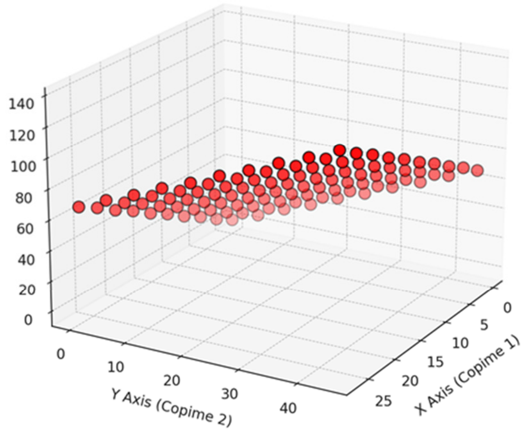

Fig. 1. The proposed 3D coprime array configuration.

A. Case 1: Four Uncorrelated Signals

This scenario evaluated the performance of the proposed algorithm in the detection of four uncorrelated signals with DOAs at angles $\theta_1 = -30°$, $\theta_2 = 15°$, $\theta_3 = 45°$, and $\theta_4 = 60°$. RMSE was computed as a function of SNR varying from -20 dB to 20 dB. This evaluation was conducted using 500 Monte Carlo trials and 100 snapshots. The results are shown in Figure 2, demonstrating the algorithm's ability to accurately resolve the DOAs of uncorrelated signals across the specified SNR range. The findings in Figure 2 highlight the algorithm's superior capability to distinguish multiple sources, showing consistent accuracy in DOA estimation for uncorrelated signals. Compared to existing methods such as 3D-Unitary Root-MUSIC [1], the modified Root-MUSIC algorithm [2], ECA-MURE [4], and FBSS [12], this approach exhibits significantly improved performance. This enhancement can be attributed to the optimized exploitation of the 3D coprime array structure, which substantially improves source separation and spatial resolution.

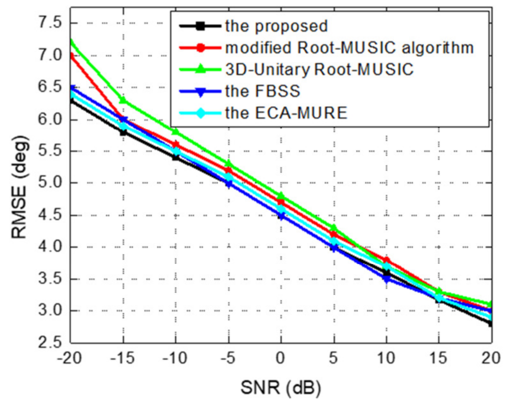

Fig. 2. RMSE versus SNR for the DOA estimation of four signals that were not correlated, with angles of $\theta_1 = -30°$, $\theta_2 = 15°$, $\theta_3 = 45°$, and $\theta_4 = 60°$.

Additionally, the use of a novel 3D joint diagonalization framework facilitates efficient signal decorrelation through the construction of a full-rank Toeplitz matrix, ensuring precise DOA estimation for uncorrelated sources. The robustness of the algorithm in practical applications is further strengthened by incorporating a newly designed cost function, which remains effective regardless of the number of sources.

B. Case 2: Four Coherent Sources

This scenario analyzed the performance of the proposed algorithm for estimating the DOAs of four coherent sources located at angles $\theta_1 = -10°$, $\theta_2 = 0°$, $\theta_3 = 10°$, and $\theta_4 = 20°$. RMSE was calculated as a function of SNR varying from -20 dB to 20 dB, using 500 Monte Carlo trials and 100 snapshots. The results, presented in Figure 3, highlight the challenges associated with accurately estimating the DOAs of coherent sources. Despite these challenges, the proposed method outperformed established techniques, including 3D-Unitary Root-MUSIC [1], the modified Root-MUSIC algorithm [2], ECA-MURE [4], and FBSS [12]. This improvement can be attributed to the effective utilization of the 3D coprime array structure, which significantly enhances source separation and spatial resolution, addressing the inherent difficulties in DOA estimation for coherent signals.

A key innovation of the proposed approach is the introduction of a novel 3D joint diagonalization framework that successfully decorrelates coherent sources by constructing a full-rank Toeplitz matrix. This process ensures more precise DOA estimation, even under challenging conditions. Furthermore, the algorithm incorporates a newly developed cost function that remains robust and independent of the number of sources, further strengthening its reliability in complex environments. The combination of the 3D coprime array structure, the joint diagonalization framework, and the resilient cost function establishes the proposed algorithm as a robust and effective solution for DOA estimation of coherent signals in diverse scenarios.

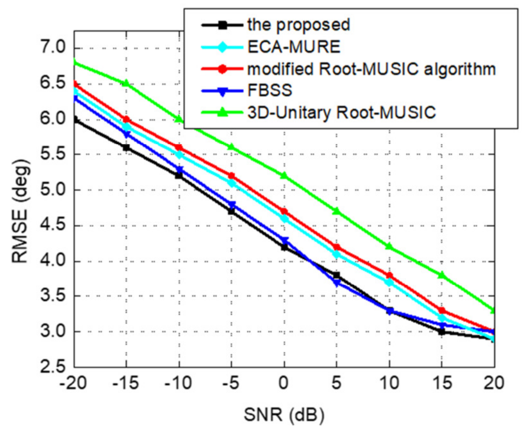

Fig. 3. RMSE versus SNR for the estimation of DOAs for four coherent sources positioned at angles $\theta_1 = -10°$, $\theta_2 = 0°$, $\theta_3 = 10°$, and $\theta_4 = 20°$.

C. Simultaneous Presence of Coherent and Uncorrelated Sources

This scenario investigated the estimation of DOAs in a challenging environment containing both coherent and uncorrelated sources. The coherent sources were located at angles $\theta_{1,1} = -20°$ and $\theta_{1,2} = 0°$, while the uncorrelated sources were positioned at $\theta_3 = 15°$ and $\theta_4 = 45°$. RMSE was computed as a function of the SNR, ranging from -20 dB to 20 dB, using 500 Monte Carlo trials and 100 snapshots. The results shown in Figure 4 highlight the complexities of accurately predicting DOAs in the presence of mixed sources, underscoring the need for advanced signal processing techniques. The proposed algorithm demonstrates superior performance in resolving both coherent and uncorrelated sources compared to established methods such as 3D-Unitary Root-MUSIC [1], the modified Root-MUSIC algorithm [2], ECA-MURE [4], and FBSS [12]. This improvement is achieved through the effective utilization of a 3D coprime array structure, which enhances source separation and spatial resolution, enabling precise DOA estimation for mixed sources. This approach leverages the 3D array design to overcome the inherent challenges posed by the coexistence of coherent and uncorrelated signals, ensuring robust and accurate DOA estimation. By integrating advanced processing techniques with an optimized array structure, the algorithm achieves significant advancements in performance, positioning it as a reliable solution for mixed-source scenarios in diverse application environments. The proposed algorithm efficiently decorrelates coherent sources while maintaining the accuracy of uncorrelated signals through the integration of a full-rank Toeplitz matrix and a novel 3D joint diagonalization framework. Additionally, the algorithm's robustness in practical scenarios is strengthened by incorporating a new cost function that operates independently of the number of sources.

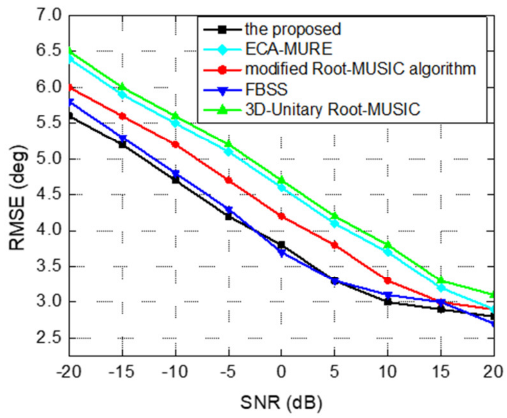

Fig. 4. RMSE versus SNR.

Figure 5 presents the RMSE as a function of the number of snapshots for estimating the DOAs of coherent sources located at $\theta_{1,1} = -20°$ and $\theta_{1,2} = 0°$ and uncorrelated sources positioned at $\theta_3 = 15°$ and $\theta_4 = 45°$. In this analysis, all parameters remain consistent with previous cases, except for the SNR, which is fixed at 10 dB. The results demonstrate the proposed method's superior ability to resolve both coherent and uncorrelated sources compared to established techniques such as 3D-Unitary Root-MUSIC [1], the modified Root-MUSIC algorithm [2], ECA-MURE [4], and FBSS [12]. This improvement is due to the efficient use of the 3D coprime array structure, which facilitates precise DOA estimation, enhances source separation, and significantly improves spatial resolution.

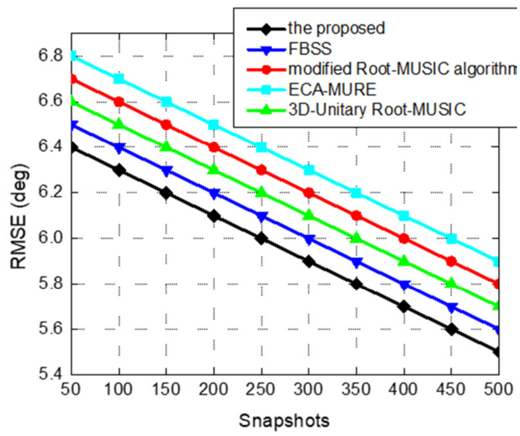

Fig. 5. RMSE versus snapshots.

By leveraging advanced array design and robust signal processing frameworks, the proposed approach proves to be a highly effective solution for mixed-source scenarios, showcasing its practicality and reliability in real-world applications.

IV. CONCLUSION

This study presents a novel approach for simultaneous DOA estimation of coherent and non-coherent sources using a 3D coprime array structure. By introducing a joint diagonalization framework that employs a full-rank Toeplitz matrix, the proposed method effectively decorrelates coherent signals while maintaining accurate estimates for uncorrelated sources. The innovative cost function, designed to be independent of the number of sources, enhances the algorithm's robustness in complex environments. Extensive simulations demonstrated that the proposed approach outperforms existing techniques, including the 3D-Unitary Root-MUSIC, modified Root-MUSIC, ECA-MURE, and FBSS algorithms. The results indicate significant improvements in accuracy and resolution, particularly in challenging scenarios involving mixed sources. The findings suggest that this advanced DOA estimation technique is well-suited for practical applications in various fields, including UAVs, radar systems, and next-generation communication networks. The capacity to accurately estimate DOA in the presence of both coherent and non-coherent sources marks a significant advancement in the field of array signal processing, paving the way for future research and development in this domain.

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