Painting Training Based Optimization: A New Human-based Metaheuristic Algorithm for Solving Engineering Optimization Problems

Syed Umar Amin
College of Computer and Information Sciences, Prince Sultan University, Riyadh, Saudi Arabia | Automated Systems and Soft Computing Lab (ASSCL), Prince Sultan University, Riyadh, Saudi Arabia
samin@psu.edu.sa

Mohammad Dehghani
Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz, Iran
m.dehghani@sutech.ac.ir (corresponding author)

Received: 12 December 2024 | Revised: 5 January 2025, 29 January 2025, and 11 February 2025 | Accepted: 14 February 2025

 DOI: https://doi.org/10.48084/etasr.9917

ABSTRACT

This study introduces a completely different perspective on optimization through the development of a novel human-based metaheuristic algorithm named Painting Training Based Optimization (PTBO). Inspired by the intricate and iterative human activities observed during painting training, PTBO models these creative and systematic processes to effectively address optimization challenges. The algorithm's foundation is rooted in the concepts of exploration and exploitation, which are essential for achieving a balance between searching the solution space widely and refining promising areas. The theoretical framework of PTBO is comprehensively described, followed by detailed mathematical modeling of its two-phase operation. To evaluate its capability, the algorithm is tested on 22 constrained optimization problems sourced from the well-regarded CEC 2011 test suite. The experimental results show that PTBO excels at producing competitive and high-quality solutions. A comparative analysis with 12 other well-known metaheuristic algorithms underscores PTBO's superior performance, particularly in handling complex benchmark functions. The results show that the proposed PTBO approach outperformed competing algorithms in all (22) optimization problems of the CEC 2011 test suite. The findings highlight PTBO's effectiveness in solving real-world optimization problems, showcasing its potential to outperform existing methods. By offering a completely different optimization approach, PTBO contributes a significant and innovative tool to address challenges in engineering and other applied domains.

Keywords-optimization; human-based metaheuristic; training instructor; Painting Training Based Optimization; exploration; exploitation

I. INTRODUCTION

Optimization is essential to address numerous scientific and real-world problems by identifying the best feasible solution among many. These problems, known as optimization problems, involve decision variables, constraints, and objective functions [1]. The goal is to determine optimal values for the decision variables while satisfying the constraints to maximize or minimize the objective function [2]. Stochastic methods, particularly metaheuristic algorithms, have gained prominence for addressing complex optimization challenges [3]. These algorithms operate through random search and trial-and-error strategies, offering flexibility and independence from problem types [4]. Metaheuristic algorithms are well-suited for non-linear, non-convex, discontinuous, and high-dimensional problems [5]. Metaheuristic algorithms are used in various applications, such as agricultural networks [6], electrical engineering [7], power grids [8], and photovoltaics [9].

A successful metaheuristic approach must balance exploration (global search) and exploitation (local search). Exploration ensures a wide search of the solution space to avoid local optima, while exploitation refines promising regions to approach the global optimum [10]. Recently published metaheuristic algorithms that can be used in various optimization applications include Potter [11], Carpet weaving [12], Sales training-based [13], Fossa [14], Addax [15], Dollmaker [16], Spider-tailed horned viper [17], Tailor [18], Orangutan [19], and Sculptor [20]. Despite the abundance of metaheuristic algorithms, they often suffer from common limitations, such as premature convergence, poor balance between exploration and exploitation, and sensitivity to parameter settings. The No Free Lunch (NFL) theorem further reinforces the fact that no single metaheuristic algorithm can consistently outperform others across all optimization problems [21]. These challenges highlight the need for continuous innovation in metaheuristic algorithms to improve performance, robustness, and adaptability. To address these limitations, this study introduces a novel algorithm, called Painting Training-Based Optimization (PTBO), designed to provide a more effective balance between exploration and exploitation while ensuring enhanced search efficiency.

Based on the best knowledge obtained from the literature review, no metaheuristic algorithm has been designed based on the simulation of the painting training process. The training activities and interactions between the instructor and painting students are an intelligent process that has a special potential to design a new optimizer. To address this research gap in the study of metaheuristic algorithms, a new metaheuristic algorithm was designed based on the mathematical modeling of the painting training process. The key contributions of this study are as follows:

- The fundamental inspiration of PTBO is human activities of (i) training applicants by the instructor and (ii) applicants' effort to improve their painting skills through practice.
- The steps of PTBO are described and then mathematically modeled in two phases: exploration and exploitation.
- The performance of PTBO was compared with the performance of 12 well-known metaheuristic algorithms.
- To assess its effectiveness in real-world applications, PTBO was applied to 22 constrained optimization problems from the CEC 2011 test suite.

II. PAINTING TRAINING-BASED OPTIMIZATION

A. Initialization

PTBO is a population-based approach where each member is mathematically modeled as a candidate solution using a vector. Thus, the entire population is represented using a matrix according to (1). At the beginning of the algorithm execution, the position of each PTBO member is initialized using (2).

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,d} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i,1} & \cdots & x_{i,d} & \cdots & x_{i,m} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,d} & \cdots & x_{N,m} \end{bmatrix}_{N\times m} \quad (1)$$

$$x_{i,d} = lb_d + r \cdot (ub_d - lb_d) \quad (2)$$

where X is the PTBO population matrix, X_i is the i^{th} painting student (candidate solution), $x_{i,d}$ is its d^{th} dimension in the search space (decision variable), N is the number of painting students, m is the number of decision variables, r is a random number in the interval $[0, 1]$, and lb_d, and ub_d are the lower bound and upper bound of the d^{th} decision variable, respectively.

The quality of each painting student (candidate solution) is evaluated using the problem's objective function. The set of evaluated values for the objective function can be represented using a vector corresponding to (3):

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N\times 1} \quad (3)$$

where F is the vector of the calculated objective function and F_i is the calculated objective function based on the i^{th} painting student. In the PTBO design, the position of each population member is updated in two separate phases based on the simulation of the painting training process. Each of these update phases is introduced and modeled below.

B. Phase 1: Education (Exploration)

In the painting training process, instructors provide tailored skills to students, gradually enhancing their abilities through structured sessions. This approach results in significant changes in population members' positions within the search space, boosting the PTBO algorithm's global exploration capability.

The first phase of PTBO simulates this process, modeling interactions between instructors and students to update positions. Using (4) and (5), new positions are calculated based on training coefficients, instructor influence, and other parameters. If a new position improves the objective function value, it replaces the current position, as defined by (6), enhancing search efficiency and effectiveness.

$$k(t) = r \cdot \frac{t}{T} \quad (4)$$

$$X_i^{P1} = X_i + k(t) \cdot (I - X_i), i = 1,2, \dots, N \quad (5)$$

$$X_i = \begin{cases} X_i^{P1}, & F_i^{P1} < F_i \\ X_i, & else \end{cases} \quad (6)$$

where $k(t)$ is the training coefficient, t is the iteration counter of the algorithm, T is the maximum number of algorithm iterations, X_i^{P1} is the new suggested position of the i^{th} painting student based on the first phase of PTBO, F_i^{P1} is its objective function value, r is a random number with a normal distribution in the range of $[0,1]$, I is the training instructor, and N is the number of painting students.

C. Phase 2: Personal Skills Improvement (Exploitation)

After acquiring skills from the instructor, students refine their abilities through practice, gradually becoming more proficient. This mirrors the optimization process, where population members adjust their positions to enhance local search efficiency. In the second phase of PTBO, members' positions are updated to simulate students' skill improvement efforts. Using (7), new positions are calculated, and if they improve the objective function value, they replace previous positions per (8). This iterative process refines positions, enhancing the algorithm's exploitation capability. Consequently, PTBO achieves a more effective balance between exploration and exploitation, improving optimization performance and convergence efficiency.

$$X_i^{P2} = X_i + (1 - 2r) \cdot \frac{(ub - lb)}{t} \quad (7)$$

$$X_i = \begin{cases} X_i^{P2}, & F_i^{P2} < F_i \\ X_i, & else \end{cases} \quad (8)$$

where X_i^{P2} is the new suggested position of the i^{th} painting student based on the second phase of PTBO, F_i^{P2} is its objective function value, t is the iteration counter of the algorithm, and T is the maximum number of algorithm iterations.

D. Computational Complexity of PTBO

The preparation and initialization of PTBO has a computational complexity equal to $O(Nm)$, where N is the number of painting students and m is the number of problem variables. In each iteration of PTBO, the position of the painting student is updated in two phases. The painting students' update process has a computational complexity of $O(2NmT)$, where T is the maximum number of iterations of the algorithm. According to this, the total computational complexity of the proposed PTBO approach is $O\big(Nm(1+2T)\big)$.

III. SIMULATION STUDIES

The proposed PTBO approach was evaluated for its effectiveness in addressing real-world optimization challenges. The CEC 2011 test suite, consisting of 22 constrained optimization problems based on practical applications, served as the benchmark for this evaluation. A detailed description of the CEC 2011 test suite is available in [22]. The titles of these real-world applications are parameter estimation for frequency-modulated (FM) sound waves, Lennard-Jones potential problem, the bifunctional catalyst blend optimal control problem, optimal control of a non-linear stirred tank reactor, tersoff potential for model Si (B), tersoff potential for model Si (C), spread spectrum radar polly phase code design, Transmission Network Expansion Planning (TNEP) problem, large scale transmission pricing problem, circular antenna array design problem, the ELD problems (consisting of: DED instance 1, DED instance 2, ELD Instance 1, ELD Instance 2, ELD Instance 3, ELD Instance 4, ELD Instance 5, hydrothermal scheduling instance 1, hydrothermal scheduling instance 2, hydrothermal scheduling instance 3), Messenger spacecraft trajectory optimization problem, and Cassini 2 spacecraft trajectory optimization problem. The performance of PTBO was compared with 12 well-established metaheuristic algorithms, including Genetic Algorithm (GA) [23], Particle Swarm Optimization (PSO) [24], Gravitational Search Algorithm (GSA) [25], Teaching–Learning-Based Optimization (TLBO) [26], Harris Hawk Optimization [27], Grey Wolf Optimizer (GWO) [28], Artificial Bee Colony (ABC) [29], Marine Predators Algorithm (MPA) [30], Tunicate Swarm Algorithm (TSA) [31], Reptile Search Algorithm (RSA) [32], African Vultures Optimization Algorithm (AVOA) [33], and White Shark Optimizer (WSO) [34].

The reasons for choosing these 12 algorithms are as follows. GA and PSO are among the most famous and first metaheuristic algorithms. GSA, TLBO, ABC, GWO, and HHO are among the most cited metaheuristic algorithms that have been used in various optimization applications. The MPA, TSA, RSA, AVOA, and WSO approaches are among the recently published successful metaheuristic algorithms that have attracted the attention of many researchers. Comparing the proposed PTBO approach with these 12 metaheuristic algorithms is a valuable competition to test the efficiency of PTBO. Simulations were implemented in MATLAB R2022a using a 64-bit Core i7 CPU with 3.20 GHz and 16 GB of main memory. The proposed PTBO approach and each of the competitor algorithms were implemented on the CEC-2011 functions in 25 independent implementations, where each implementation contains 150,000 Function Evaluations (FEs). The simulation results involved three statistical indicators: mean, standard deviation (std), and Execution Time (ET). Table I presents the comparison of the PTBO algorithm with its competitors on the CEC 2011 test suite, offering a clear summary of the results. In addition, the performance of PTBO and competitor algorithms in handling some functions of the CEC 2011 test suite is shown as boxplots in Figure 1 and as convergence curves in Figure 2.

The simulation results demonstrate that PTBO consistently outperformed all other algorithms across all test problems (C11-F1 to C11-F22), highlighting its robustness and adaptability to tackle complex real-world optimization challenges. PTBO ranks as the top optimizer in most problems within the CEC 2011 test suite, affirming its effectiveness. Additionally, statistical validation using the Wilcoxon rank-sum test [35] confirms the significant superiority of PTBO over competing algorithms. This statistical evidence reinforces PTBO's ability to efficiently optimize challenging problems. Overall, PTBO's strong performance and validation underscore its potential as a powerful and reliable optimization tool.

A. PTBO for Practical Applications

A real-world challenge, called pressure vessel design problem, was selected to analyze the performance of PTBO to address practical applications. The pressure vessel design problem is a real-world engineering application to minimize construction costs. Figure 3 presents the schematic of this design. The pressure vessel design mathematical model is as follows [36]:

Consider: $X = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L]$.

Minimize:

$$f(x) = 0.6224x_1x_3x_4 + 1.778x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3.$$

Subject to:

$$g_1(x) = -x_1 + 0.0193x_3 \le 0,$$

$$g_2(x) = -x_2 + 0.00954x_3 \le 0,$$

$$g_3(x) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0,$$

$$g_4(x) = x_4 - 240 \le 0.$$

With:

$$0 \le x_1, x_2 \le 100 \text{ and } 10 \le x_3, x_4 \le 200.$$

Table II presents the implementation results of PTBO and competitor algorithms in solving the pressure vessel design.

TABLE I. PERFORMANCE OF METAHEURISTIC ALGORITHMS ON CEC 2011 TEST SUITE

		PTBO	WSO	AVOA	RSA	MPA	TSA	ABC	HHO	GWO	TLBO	GSA	PSO	GA
C11-F1	mean	**5.920103**	18.86534	13.66405	23.57116	7.732586	19.66976	10.15203	9.330005	11.35755	19.70491	23.26431	19.16242	25.12702
	std	7.196379	2.301772	4.455172	1.817054	5.888225	**1.095768**	5.243586	4.702153	7.850972	1.945439	1.412726	7.161112	1.353343
	E.T	**0.621113**	1.147059	1.08202	1.30252	1.685247	0.737185	0.766996	0.991598	0.752646	2.475677	1.699408	0.786887	0.790768
C11-F2	mean	**-26.3179**	-13.3376	-20.5808	-10.272	-24.9832	-9.96649	-22.4716	-21.3055	-22.3139	-9.53874	-14.5934	-22.3671	-11.7511
	std	0.738935	1.536105	0.641921	**0.527779**	1.016521	3.257691	1.165737	1.177602	2.875701	1.010239	4.733202	1.897831	2.199001
	E.T	**1.219837**	3.122641	2.116327	4.733408	2.995567	1.435136	1.401353	1.511874	1.526617	4.026616	3.163467	1.308459	1.347199
C11-F3	mean	**1.15E-05**	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05
	std	2E-19	2.38E-11	2.73E-09	5.36E-11	1.33E-15	2.56E-14	1.34E-15	2.14E-13	4.01E-15	8.42E-14	2.15E-19	**6.26E-20**	2.97E-18
	E.T	743.8869	596.2613	875.0575	585.0686	1217.047	609.0948	607.9165	607.454	608.6231	1748.186	818.5322	**575.8977**	617.9594
C11-F4	mean	**21.62299**	47.6873	31.4835	61.06053	26.29892	36.35472	33.08964	28.35652	30.72392	92.1311	47.82484	95.30975	84.22624
	std	2.956008	4.061216	3.629764	9.731766	2.218548	3.186096	2.381759	**1.409536**	1.908046	19.52532	6.978454	11.47775	1.905813
	E.T	15.07985	18.59457	20.18058	18.81958	30.37307	16.72845	16.36406	18.36957	14.71633	39.0127	12.24346	14.55213	**11.4772**
C11-F5	mean	**-34.1274**	-24.0753	-27.6415	-18.8351	-33.2106	-26.587	-31.1101	-31.764	-31.377	-8.89625	-26.8212	-6.55518	-7.48518
	std	0.589989	1.036006	0.792793	2.677522	1.005792	4.600593	0.98161	**0.339155**	3.229141	1.864536	3.624792	2.838385	1.571807
	E.T	**17.66267**	21.63869	26.67527	23.08973	40.65085	20.24999	19.94252	20.18764	20.07313	56.89843	20.81698	23.56863	19.47957
C11-F6	mean	**-24.1119**	-13.2358	-18.6357	-12.1609	-22.5025	-6.23132	-21.1586	-19.5135	-19.2836	-0.56633	-21.7188	-1.50272	-2.48319
	std	2.324951	0.528541	1.556913	0.932023	2.222076	6.870454	2.475545	1.893858	2.238971	**0.054608**	4.439004	2.02309	4.01175
	E.T	**13.60279**	21.84725	25.57683	23.08044	41.60155	20.42178	20.71275	20.61406	20.66935	58.46636	21.3468	24.64162	19.49718
C11-F7	mean	**0.860699**	1.661227	1.315385	1.998272	0.934739	1.334548	1.219219	0.931711	1.082821	1.782185	1.095763	1.142938	1.805564
	std	0.211503	0.076276	0.161037	0.203321	0.119058	0.269329	**0.056414**	0.086386	0.230746	0.16399	0.195858	0.323976	0.289565
	E.T	**1.204608**	2.618364	2.178151	3.572109	3.413572	1.623746	1.513652	1.66462	1.532956	4.603534	1.91725	1.702331	1.42789
C11-F8	mean	**220**	290.0355	242.0692	333.5928	222.6366	260.2079	237.5333	223.2518	227.9098	224.3943	248.4028	488.9614	222.6854
	std	**0**	30.37888	16.43115	39.78566	3.200037	73.85693	12.76309	2.358784	9.600111	9.237711	39.42551	172.6462	5.645268
	E.T	**3.190836**	4.394774	5.18949	4.784833	8.056244	3.951239	3.889735	4.004715	3.940096	11.53115	4.247251	4.428498	3.938002
C11-F9	mean	**8789.286**	600510.3	407522.9	1145069	21100.06	70758.76	139533.5	46752.24	45721.6	440204.5	887581.9	1167146	2095233
	std	**3889.181**	143461.1	36104.54	284331.5	8858.822	17497.04	68240.93	16690.07	27265.46	92690.1	92016.76	276865.9	108773.3
	E.T	**8.138154**	18.94226	15.4447	26.02624	23.28378	11.61257	11.01663	11.85829	11.5064	32.24512	13.08107	13.25627	10.74148
C11-F10	mean	**-21.4889**	-13.3884	-16.568	-11.565	-18.8367	-13.8405	-16.0518	-17.6395	-13.5323	-10.4964	-12.508	-10.6039	-10.2854
	std	0.498616	0.937062	0.260797	0.308607	0.424949	3.483813	0.28676	1.228437	0.890177	0.066255	0.70255	0.031959	**0.025285**
	E.T	21.08799	22.56117	29.06124	22.99103	43.64193	21.9544	21.68244	22.13131	21.43308	61.8121	21.51456	**20.48358**	20.6292
C11-F11	mean	**571712.3**	6208039	1023365	9502231	1743127	6361469	1951149	1784685	4085956	5569495	1476053	5581432	6553225
	std	260922.1	321019	177006.8	216635.2	118949.9	1040504	98143.66	156706.5	292595	**6128.444**	144405.8	13127.11	58793.41
	E.T	**3.396937**	10.75344	8.078823	17.17506	8.026048	3.913679	13.50512	4.331815	4.005783	10.27428	5.514548	3.521359	3.619429
C11-F12	mean	**1199805**	8947435	3543848	14167436	1280336	5325909	2771991	1299785	1441398	15345024	6144876	2400042	15518397
	std	**47157.58**	305529.5	81136.19	819570.5	73470.4	220454.3	87414.67	59400.18	139313.3	712864	244963.2	179936	117514.3
	E.T	**3.793065**	22.17211	14.43135	36.57603	14.09973	7.279686	26.13292	7.768194	7.321105	18.25782	10.3244	6.274536	6.328642
C11-F13	mean	**15444.2**	15889.92	15448.29	16377.96	15464.65	15493.78	15494.17	15476.34	15505.55	15971.37	137154.5	15494.47	31139.66
	std	**0.009091**	342.796	1.004178	787.4918	3.164156	12.62191	15.01561	5.091162	9.494873	445.5917	42749.3	27.88738	32692.62
	E.T	**0.606276**	1.101786	1.136469	1.473014	1.430873	0.722044	1.266859	0.730559	0.680644	2.216298	0.972618	0.681876	0.713066
C11-F14	mean	**18295.35**	119857.7	18536.91	245609.7	18632.25	19628.09	18933.22	18840.97	19306.06	333721.4	19155.14	19190.25	19176.5
	std	**71.59938**	36381.65	109.3914	81968.31	74.39378	421.1453	104.1157	77.81174	165.5597	309983.9	245.1959	141.7687	272.1121
	E.T	**0.427157**	1.326322	1.053172	2.07299	1.127994	0.553375	1.603479	0.579655	0.515422	1.60509	0.890782	0.497711	0.507926
C11-F15	mean	**32883.58**	977284.1	113766.5	2063593	32953.46	56265.61	93075.75	32993.17	33093.21	16636937	320869.9	33319.99	8561059
	std	76.94696	1043706	83527.2	2335201	63.16742	48566.13	43023.5	56.91084	48.81009	10192827	30627.71	**8.323452**	5194658
	E.T	**0.565858**	1.57669	1.367453	2.30257	1.666847	0.827405	2.011513	0.85081	0.784439	2.276938	1.166119	0.731841	0.761925
C11-F16	mean	**133550**	1009312	135305.4	2092804	137981.6	146039.9	140769.5	139320.4	146844.8	95917046	20191734	85849182	82429084
	std	2392.2	991512	**1065.32**	2229326	2742.079	2577.48	3336.55	2121.618	4097.123	2295333	11948669	14306297	17331710
	E.T	**0.433626**	2.880442	1.842664	4.868135	1.597066	0.731681	3.918149	0.892749	0.743205	1.834396	1.395037	0.553988	0.56492
C11-F17	mean	**1926615**	9.67E+09	2.5E+09	1.67E+10	2319740	1.38E+09	3.14E+09	2536263	3105990	2.41E+10	1.21E+10	2.25E+10	2.36E+10
	std	**12003.53**	1.15E+09	2.15E+08	3.81E+09	483234.6	2.38E+08	8.55E+08	387183.8	1452554	8.54E+08	1.04E+09	2.92E+09	2.19E+09
	E.T	**1.727172**	10.82082	7.05646	18.59715	5.198236	2.575721	14.0566	3.044497	2.670038	7.715739	4.519795	2.566988	2.630842
C11-F18	mean	**942057.5**	59259731	6999338	1.28E+08	974093.8	2137788	3770312	980829.7	1038166	33379146	11933688	1.45E+08	1.24E+08
	std	**2774.139**	13134468	3856605	28361360	44024.51	328191.1	1864223	35754.35	128228.5	4881764	2905402	18542586	3905050
	E.T	**1.678574**	7.954503	5.727238	12.92761	5.425479	2.665727	11.02835	2.981397	2.831208	6.959598	4.026526	2.3726	2.426632
C11-F19	mean	**1025341**	58323230	7101979	1.25E+08	1146721	2577158	4122770	1231987	1389482	38326178	6682003	1.86E+08	1.24E+08
	std	**99675.04**	11585725	1074405	24108049	110578.5	347987.2	2675275	116004.1	141440.2	9568316	3016064	21150583	2917747
	E.T	**2.354316**	9.401411	6.991849	14.45363	7.592415	3.778641	11.9253	4.349197	3.795235	9.93465	5.329499	3.338571	3.43572
C11-F20	mean	**941250.4**	62025771	6284195	1.35E+08	961857.1	1896577	3014281	966608.4	1003070	37232572	15325514	1.72E+08	1.24E+08
	std	5013.552	8466023	679054.4	19001005	**2967.269**	263543.7	143223.5	4711.274	18664.06	744740.1	6251347	17309602	4691554
	E.T	**2.231574**	9.718457	6.912618	15.13121	7.498915	3.893597	11.4622	4.305707	3.855959	9.956753	5.153071	3.370197	3.421131
C11-F21	mean	**12.71443**	53.22814	22.35281	81.52848	16.19137	31.18466	24.63611	19.04394	23.14526	107.6811	42.92665	113.116	109.738
	std	2.412667	9.197906	1.365271	19.79151	2.164909	2.573762	**0.489513**	2.02894	1.921484	46.54809	4.028964	14.70625	35.23633
	E.T	58.27956	28.15006	116.7231	28.47101	51.89277	25.96351	25.78215	27.82912	25.74904	433.5071	26.45769	**24.85448**	324.6405
C11-F22	mean	**16.12513**	49.25857	28.34713	67.25667	19.31665	33.4131	29.10109	22.4888	25.69968	109.4977	49.1256	113.852	98.66914
	std	4.197797	5.711511	5.338647	13.75259	2.413637	2.993295	2.741252	2.226558	1.277059	27.9599	7.921328	14.30381	**1.097372**
	E.T	23.87322	24.9927	85.603	25.70583	47.58784	23.85285	23.52593	24.27957	23.53612	348.8059	24.34236	**22.87887**	272.1996
Sum rank		**22**	204	131	243	53	164	125	84	115	236	172	216	237
Mean rank		**1**	9.272727	5.954545	11.04545	2.409091	7.454545	5.681818	3.818182	5.227273	10.72727	7.818182	9.818182	10.77273
Total rank		**1**	9	6	13	2	7	5	3	4	11	8	10	12
Wilcoxon: *p*-value			2.00E-14	3.54E-17	7.12E-18	7.97E-06	2.23E-17	6.25E-16	2.38E-13	8.79E-15	1.52E-17	3.67E-17	7.12E-18	1.04E-17

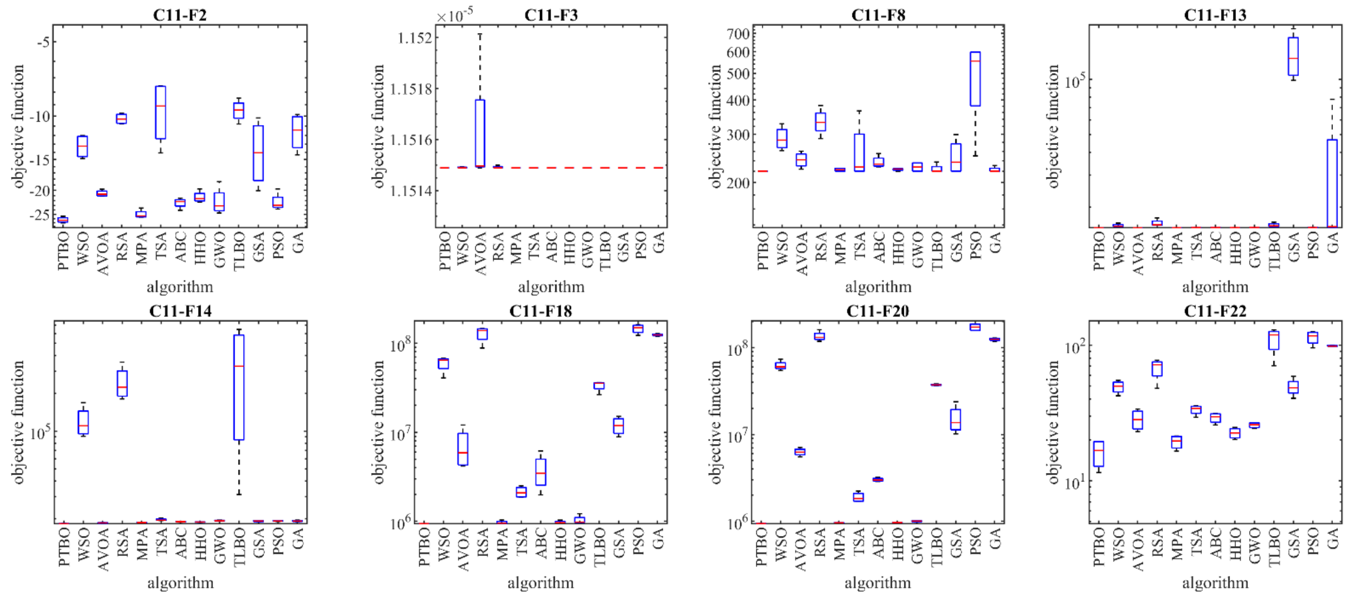

Fig. 1. Boxplot diagrams of PTBO and competitor algorithms performances on CEC 2011 test suite.

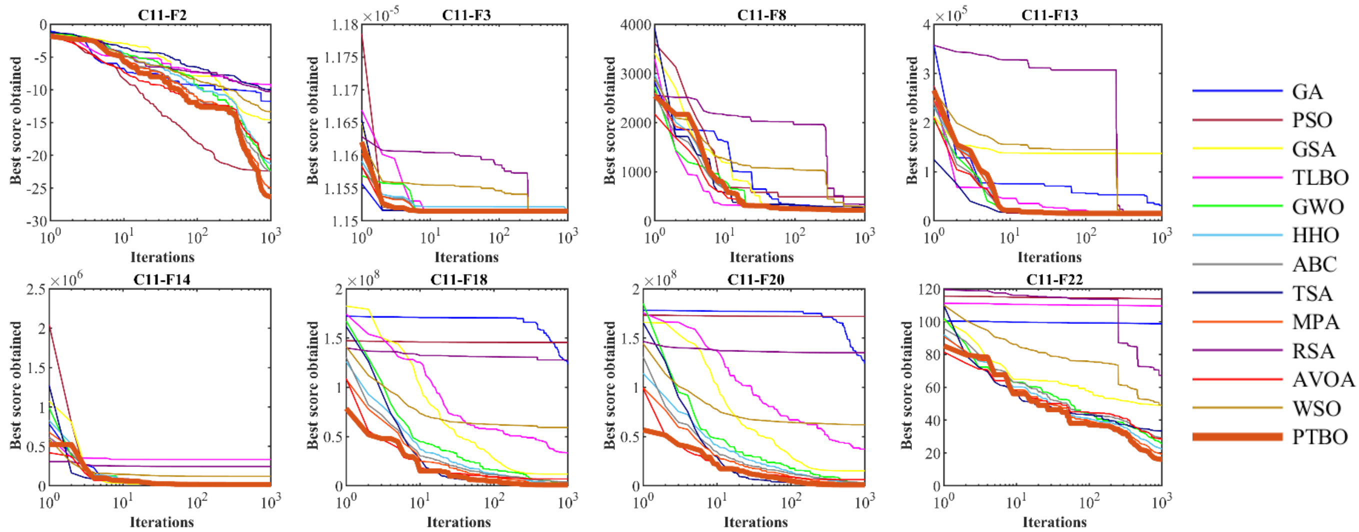

Fig. 2. Convergence curves of PTBO and competitor algorithms on CEC 2011 test suite.

TABLE II. PERFORMANCE OF METAHEURISTIC ALGORITHMS ON PRESSURE VESSEL DESIGN PROBLEM

Algorithm	Optimum variables					Statistical results					
	T_s	T_h	R	L		mean	best	worst	std	median	rank
PTBO	0.7781686	0.3846492	40.319619	200		5885.3269	5885.3269	5885.3269	2.32E-08	5885.3269	1
WSO	0.7781686	0.3846492	40.319619	200		5907.0116	5885.3328	6094.6066	53.104718	5885.3328	3
AVOA	0.7781903	0.3846599	40.320741	199.98438		6417.9548	5885.3699	7301.8994	485.20832	6249.9212	6
RSA	0.8538833	0.4168324	40.384828	200		12102.459	6547.244	20969.984	3923.608	11268.436	9
MPA	0.7781686	0.3846492	40.319619	200		5885.3328	5885.3328	5885.3328	3.91E-06	5885.3328	2
TSA	0.7797577	0.3858656	40.396543	200		6259.4574	5913.0272	7323.2575	391.18422	6101.1243	5
HHO	0.8128458	0.5410129	40.396428	198.93353		7978.5287	6581.1487	12433.243	1390.3396	7795.4782	8
ABC	0.8182023	0.4061992	42.35271	173.53517		6576.3826	5968.7277	7273.5051	448.12811	6572.6466	7
GWO	0.778454	0.3856252	40.327164	199.9429		5945.5249	5890.2372	6636.6949	163.66399	5901.7579	4
TLBO	1.1978846	1.2639943	61.056155	91.741588		39032.938	14709.572	69674.581	15506.905	38454.342	12
GSA	0.9570181	0.4737273	49.581737	144.99986		24592.051	7674.4951	39531.961	8743.4838	26413.078	10
PSO	1.2767681	2.3221527	50.647022	110.15344		41177.001	17231.344	89983.884	18842.419	38677.476	13
GA	1.1434316	0.7799386	54.784772	96.515001		29575.454	9745.9423	60485.678	14026.271	26621.06	11

PTBO provided the optimal solution for this design, with design variable values equal to 0.7781686, 0.3846492, 40.319619, 200, and the corresponding objective function value equal to 5885.3269. Figure 4 shows the convergence curve of PTBO while achieving the solution for pressure vessel design. Based on the simulation results, the proposed PTBO provided superior performance in pressure vessel design optimization compared to competitor algorithms.

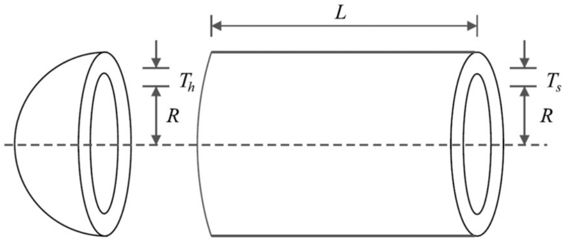

Fig. 3. Schematic of pressure vessel design.

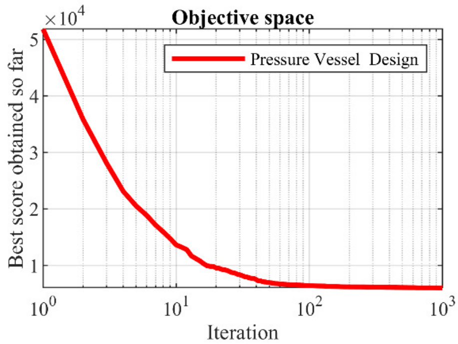

Fig. 4. PTBO's performance convergence curve on pressure vessel design.

B. Theoretical Justification and Discussion

The PTBO algorithm demonstrates theoretical superiority through its structural principles, divided into three key aspects:

- Two-Phase Structure: PTBO employs distinct exploration (training phase) and exploitation (individual improvement phase) stages. The training phase performs a broad global search, mimicking a human learning process, to identify promising solution regions while avoiding premature convergence. The improvement phase refines these regions, akin to a student practicing skills, ensuring incremental progress toward optimal solutions without overfitting.

- Instructor-Student Interaction: A unique feature of PTBO is the dynamic collaboration between instructor and students, where the instructor facilitates significant adjustments in the search space, guiding students' gradual improvements. This interaction prevents entrapment in local optima, particularly in complex spaces, by balancing wide-scale exploration with focused refinement.

- Dynamic Balancing: PTBO dynamically balances exploration and exploitation, emphasizing global search early and local refinement later. This strategy improves optimization efficiency, avoids local optima, and ensures robust performance in complex spaces.

C. Population Diversity, Exploration, and Exploitation Analysis

Population diversity in PTBO describes the spatial distribution of individuals within the search space, playing a vital role in monitoring the algorithm's search behavior. This metric highlights whether the population is oriented towards exploring new solutions or refining existing ones. By analyzing the diversity within the PTBO population, it becomes possible to evaluate and refine the algorithm's capacity for exploration and exploitation as a collective. Various definitions of diversity have been proposed in the literature. In [37], the concept of diversity was introduced using the following equations:

$$Diversity = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\sum_{d=1}^{m}\left(x_{i,d}-\bar{x}_d\right)^2} \tag{9}$$

$$\bar{x}_d = \frac{1}{N}\sum_{i=1}^{N}x_{i,d} \tag{10}$$

where N represents the number of population members, m is the number of problem dimensions, and $\bar{x}_d$ is the mean of the population in the d^{th} dimension. Therefore, the extent of exploration and exploitation within the population for each iteration can be defined by:

$$Exploration = \frac{Diversity}{Diversity_{max}}, \tag{11}$$

$$Exploitation = \frac{|Diversity - Diversity_{max}|}{Diversity_{max}} \tag{12}$$

Table III summarizes the findings on population diversity, exploration, and exploitation.

TABLE III. POPULATION DIVERSITY, EXPLORATION, AND EXPLOITATION PERCENTAGE RESULTS

Function	Exploitation	Exploration	Diversity	
			Last iteration	First iteration
C11-F1	1	9.39E-165	1.22E-162	129.60658
C11-F2	1	0	0	17.253897
C11-F3	1	0	0	264.31188
C11-F4	1	0	0	211.89513
C11-F5	1	0	0	39.256078
C11-F6	0.9876403	0.0123597	1.4567729	117.86504
C11-F7	0.927536	0.072464	0.1004623	1.386376
C11-F8	1	5.96E-10	1.23E-06	1290.0798
C11-F9	1	4.04E-10	4.23E-09	10.449092
C11-F10	1	1.70E-17	7.82E-16	46.046617
C11-F11	1	3.61E-11	2.64E-08	730.84902
C11-F12	1	0	0	78.178087
C11-F13	1	0	0	83.89034
C11-F14	1	2.32E-09	7.54E-08	32.439964
C11-F15	1	4.37E-11	1.26E-10	2.8757232
C11-F16	1	0	0	1.6292818
C11-F17	1	1.28E-09	5.01E-09	3.9168571
C11-F18	1	2.902E-10	2.67E-10	0.9188602
C11-F19	0.7645502	0.2354498	0.1117081	0.3785558
C11-F20	0.8329345	0.1670655	0.0715234	0.4281158
C11-F21	1	8.75E-11	3.92E-10	3.661421
C11-F22	1	2.79E-10	7.70E-10	2.7633739

Figure 5 demonstrates the exploration-exploitation ratio of the PTBO method over the iteration process, providing a visual tool for understanding how the algorithm balances global and local search strategies. The simulation results reveal that PTBO effectively maintains population diversity, with higher values observed during initial iterations and lower values in later stages. Moreover, the exploration-to-exploitation ratio generally converges towards approximately 0.00%:100%, indicating that the PTBO systematically emphasizes exploitation in the latter stages of the optimization process. These results confirm that the proposed PTBO approach leverages appropriate population diversity to achieve a balance between exploration and exploitation, thereby optimizing its performance throughout the search process.

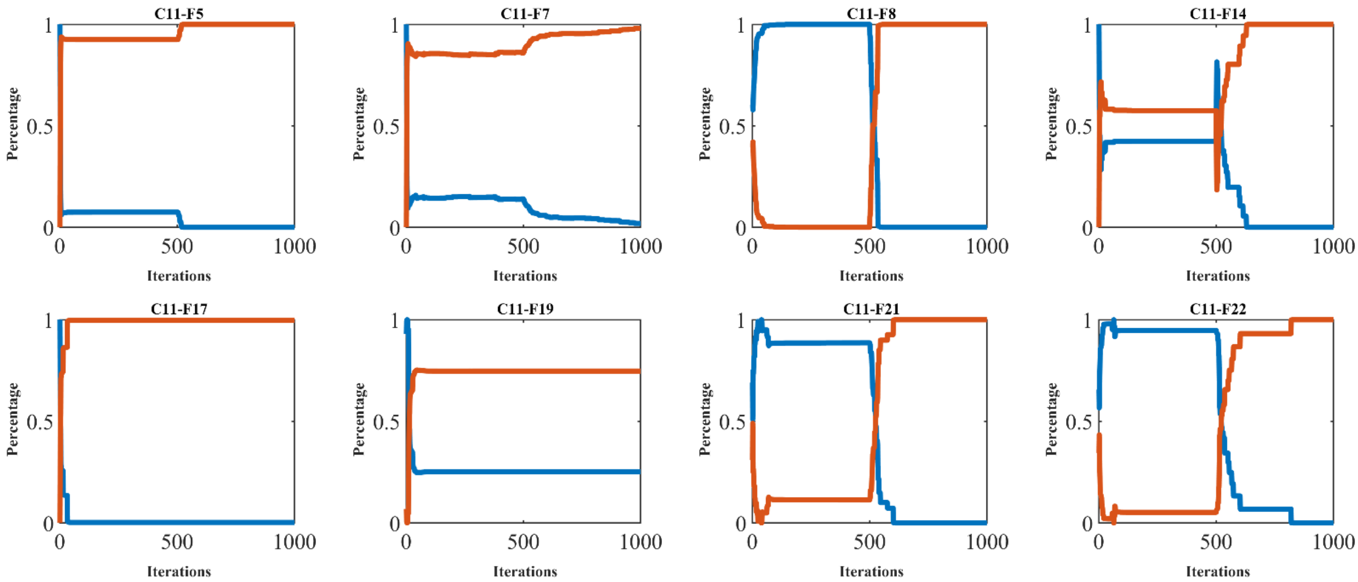

Fig. 5. Exploration and exploitation of PTBO.

D. *Sensitivity Analysis*

The sensitivity analysis of the proposed PTBO was examined for two key parameters: maximum number of iterations and population size. For this purpose, seven functions, F1 to F7, were used, whose details can be found in [38].

To assess the impact of the maximum number of iterations on the performance of PTBO, the algorithm was executed independently with iteration values of 100, 500, 800, and 1000 on the seven objective functions. The results, summarized in Table IV, reveal a clear trend: as the number of iterations increases, the algorithm exhibits enhanced convergence toward optimal solutions. This is evidenced by the decreasing values of the objective functions across all test cases. Notably, for F1 to F4, the objective values decrease exponentially as iterations increase, demonstrating PTBO's ability to refine solutions progressively. For F5, the reduction is less pronounced but still evident, suggesting that although more iterations improve the results, the impact may vary depending on the function characteristics. Functions F6 and F7 also exhibit a steady decline in objective values, reinforcing the effectiveness of PTBO in handling diverse optimization landscapes.

In the second stage, the sensitivity of PTBO to population size was investigated by running the algorithm with population sizes of 20, 30, 50, and 80 on the same set of objective functions. The results in Table V indicate that increasing the number of population members consistently improves solution quality by further reducing the objective function values. This trend is particularly evident in F1 to F4, where a larger population leads to substantial improvements in convergence. For F5, the objective values decrease as the population size grows, although the effect is more gradual. F6 maintains an optimal value of zero across all cases, confirming its stability. In F7, the results highlight a significant improvement as the population size increases, suggesting that a larger population enhances the search process and helps the algorithm avoid premature convergence.

TABLE IV. SENSITIVITY ANALYSIS ON THE MAXIMUM NUMBER OF ITERATIONS

Objective function	Maximum number of iterations			
	100	**500**	**800**	**1000**
F_1	6.00E-09	7.74E-100	4.40E-205	7.6E-260
F_2	3.49E-06	3.58E-56	5.08E-113	2.2E-142
F_3	480.5555	1.44E-07	2.08E-28	4.01E-40
F_4	0.004132	3.13E-39	2.13E-79	5E-102
F_5	28.65352	27.87766	27.87319	27.05356
F_6	0	0	0	0
F_7	0.015142	0.004397	0.003891	0.000593

TABLE V. SENSITIVITY ANALYSIS ON THE NUMBER OF POPULATION MEMBERS

Objective function	Number of population members			
	20	**30**	**50**	**80**
F_1	1.46E-240	2.20E-255	7.6E-260	4.31E-262
F_2	2.93E-140	4.30E-141	2.2E-142	4.63E-146
F_3	8.27E-29	4.14E-36	4.01E-40	2.03E-45
F_4	2.63E-100	9.23E-101	5E-102	8.06E-105
F_5	28.89621	28.12965	27.05356	26.93898
F_6	0	0	0	0
F_7	0.010739	0.005518	0.000593	0.000205

Overall, the findings of both analyses confirm that increasing the number of iterations and the population size contributes to the improved performance of PTBO. However, the extent of this improvement varies depending on the nature of the objective function, indicating that an optimal balance between these parameters must be considered to maximize efficiency.

IV. CONCLUDING REMARKS AND FUTURE WORK

This paper presents PTBO, a novel metaheuristic algorithm inspired by the painting training process, including instructor guidance and students' practice efforts. PTBO operates in two phases, exploration and exploitation, mathematically modeled to optimize the search process. Tested on the CEC 2011 suite of 22 real-world problems, PTBO demonstrated superior performance, balancing exploration and exploitation effectively. Compared to 12 established metaheuristic algorithms, PTBO achieved improved results in numerous benchmark functions, confirming its effectiveness in solving diverse optimization challenges. Despite its advantages, PTBO also has limitations. As with other stochastic methods, one of its limitations is that there is no guarantee of reaching the global optimum. Another limitation of PTBO is that it is always possible to design newer algorithms with superior performance.

The study also highlights several avenues for future work. A key direction is the development of binary and multi-objective versions of PTBO, which could enhance its versatility. Furthermore, applying PTBO to solve optimization problems in various scientific domains and real-world scenarios, such as energy management, robotic control, logistics planning, or network optimization, presents numerous opportunities for further exploration. Other research proposals include studies of PTBO applications on large-scale problems, highly nonlinear problems, or dynamic environments.

ACKNOWLEDGMENT

The authors would like to thank Prince Sultan University, Riyadh, Saudi Arabia, for funding the Article Processing Charges (APCs) of this publication. This research is supported by the Automated Systems and Soft Computing Lab (ASSCL), Prince Sultan University, Riyadh, Saudi Arabia.

REFERENCES

[1] S. Zhao, T. Zhang, S. Ma, and M. Chen, "Dandelion Optimizer: A nature-inspired metaheuristic algorithm for engineering applications," *Engineering Applications of Artificial Intelligence*, vol. 114, Sep. 2022, Art. no. 105075, https://doi.org/10.1016/j.engappai.2022.105075.

[2] X. Wang, "Draco lizard optimizer: a novel metaheuristic algorithm for global optimization problems," *Evolutionary Intelligence*, vol. 18, no. 1, Nov. 2024, Art. no. 10, https://doi.org/10.1007/s12065-024-00998-5.

[3] N. T. Linh, "A Novel Combination of Genetic Algorithm, Particle Swarm Optimization, and Teaching-Learning-Based Optimization for Distribution Network Reconfiguration in Case of Faults," *Engineering, Technology & Applied Science Research*, vol. 14, no. 1, pp. 12959–12965, Feb. 2024, https://doi.org/10.48084/etasr.6718.

[4] M. Q. Taha, M. K. Mohammed, and B. E. Haiba, "Metaheuristic Optimization of Maximum Power Point Tracking in PV Array under Partial Shading," *Engineering, Technology & Applied Science Research*, vol. 14, no. 3, pp. 14628–14633, Jun. 2024, https://doi.org/10.48084/etasr.7385.

[5] Z. Benmamoun, K. Khlie, M. Dehghani, and Y. Gherabi, "WOA: Wombat Optimization Algorithm for Solving Supply Chain Optimization Problems," *Mathematics*, vol. 12, no. 7, Jan. 2024, Art. no. 1059, https://doi.org/10.3390/math12071059.

[6] A. Rehman, I. Abunadi, K. Haseeb, T. Saba, and J. Lloret, "Intelligent and trusted metaheuristic optimization model for reliable agricultural network," *Computer Standards & Interfaces*, vol. 87, Jan. 2024, Art. no. 103768, https://doi.org/10.1016/j.csi.2023.103768.

[7] H. H. Ammar, A. T. Azar, R. Shalaby, and M. I. Mahmoud, "Metaheuristic Optimization of Fractional Order Incremental Conductance (FO-INC) Maximum Power Point Tracking (MPPT)," *Complexity*, vol. 2019, no. 1, 2019, Art. no. 7687891, https://doi.org/10.1155/2019/7687891.

[8] R. Somakumar, P. Kasinathan, G. Monicka, A. Rajagopalan, V. K. Ramachandaramurthy, and U. Subramaniam, "Optimization of emission cost and economic analysis for microgrid by considering a metaheuristic algorithm-assisted dispatch model," *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 35, no. 4, 2022, Art. no. e2993, https://doi.org/10.1002/jnm.2993.

[9] M. C. Z. Zambou, A. S. T. Kammogne, M. S. Siewe, A. T. Azar, S. Ahmed, and I. A. Hameed, "Optimized Nonlinear PID Control for Maximum Power Point Tracking in PV Systems Using Particle Swarm Optimization," *Mathematical and Computational Applications*, vol. 29, no. 5, Oct. 2024, Art. no. 88, https://doi.org/10.3390/mca29050088.

[10] Z. Benmamoun, K. Khlie, M. Dehghani, and Y. Gherabi, "WOA: Wombat Optimization Algorithm for Solving Supply Chain Optimization Problems," *Mathematics*, vol. 12, no. 7, Jan. 2024, Art. no. 1059, https://doi.org/10.3390/math12071059.

[11] T. Hamadneh *et al.*, "On the Application of Potter Optimization Algorithm for Solving Supply Chain Management Application.," *International Journal of Intelligent Engineering & Systems*, vol. 17, no. 5, 2024.

[12] S. Alomari *et al.*, "Carpet Weaver Optimization: A Novel Simple and Effective Human-Inspired Metaheuristic Algorithm.," *International Journal of Intelligent Engineering & Systems*, vol. 17, no. 4, 2024.

[13] T. Hamadneh *et al.*, "Sales Training Based Optimization: A New Human-inspired Metaheuristic Approach for Supply Chain Management.," *International Journal of Intelligent Engineering & Systems*, vol. 17, no. 6, 2024.

[14] T. Hamadneh *et al.*, "Fossa optimization algorithm: A new bio-inspired metaheuristic algorithm for engineering applications," *International Journal of Intelligent Engineering & Systems*, vol. 17, no. 5, pp. 1038–1047, 2024.

[15] T. Hamadneh *et al.*, "Addax Optimization Algorithm: A Novel Nature-Inspired Optimizer for Solving Engineering Applications.," *International Journal of Intelligent Engineering & Systems*, vol. 17, no. 3, 2024.

[16] K. Kaabneh *et al.*, "Dollmaker Optimization Algorithm: A Novel Human-Inspired Optimizer for Solving Optimization Problems.," *International Journal of Intelligent Engineering & Systems*, vol. 17, no. 3, 2024.

[17] T. Hamadneh *et al.*, "Spider-Tailed Horned Viper Optimization: An Effective Bio-Inspired Metaheuristic Algorithm for Solving Engineering Applications.," *International Journal of Intelligent Engineering & Systems*, vol. 18, no. 1, 2025.

[18] T. Hamadneh *et al.*, "On the Application of Tailor Optimization Algorithm for Solving Real-World Optimization Application.," *International Journal of Intelligent Engineering & Systems*, vol. 18, no. 1, 2025.

[19] T. Hamadneh *et al.*, "Orangutan optimization algorithm: An innovative bio-inspired metaheuristic approach for solving engineering optimization problems," *International Journal of Intelligent Engineering & Systems*, vol. 18, no. 1, 2025.

[20] T. Hamadneh *et al.*, "Sculptor Optimization Algorithm: A New Human-Inspired Metaheuristic Algorithm for Solving Optimization Problems.," *International Journal of Intelligent Engineering & Systems*, vol. 17, no. 4, 2024.

[21] D. H. Wolpert and W. G. Macready, "No free lunch theorems for optimization," *IEEE Transactions on Evolutionary Computation*, vol. 1, no. 1, pp. 67–82, Apr. 1997, https://doi.org/10.1109/4235.585893.

[22] S. Das and P. N. Suganthan, "Problem definitions and evaluation criteria for CEC 2011 competition on testing evolutionary algorithms on real world optimization problems," Jadavpur University, Nanyang Technological University, India, 2010.

[23] D. E. Goldberg and J. H. Holland, "Genetic Algorithms and Machine Learning," *Machine Learning*, vol. 3, no. 2, pp. 95–99, Oct. 1988, https://doi.org/10.1023/A:1022602019183.

[24] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proceedings of ICNN'95 - International Conference on Neural Networks*, Perth, WA, Australia, 1995, vol. 4, pp. 1942–1948, https://doi.org/10.1109/ICNN.1995.488968.

[25] E. Rashedi, H. Nezamabadi-pour, and S. Saryazdi, "GSA: A Gravitational Search Algorithm," *Information Sciences*, vol. 179, no. 13, pp. 2232–2248, Jun. 2009, https://doi.org/10.1016/j.ins.2009.03.004.

[26] R. V. Rao, V. J. Savsani, and D. P. Vakharia, "Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems," *Computer-Aided Design*, vol. 43, no. 3, pp. 303–315, Mar. 2011, https://doi.org/10.1016/j.cad.2010.12.015.

[27] A. A. Heidari, S. Mirjalili, H. Faris, I. Aljarah, M. Mafarja, and H. Chen, "Harris hawks optimization: Algorithm and applications," *Future Generation Computer Systems*, vol. 97, pp. 849–872, Aug. 2019, https://doi.org/10.1016/j.future.2019.02.028.

[28] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey Wolf Optimizer," *Advances in Engineering Software*, vol. 69, pp. 46–61, Mar. 2014, https://doi.org/10.1016/j.advengsoft.2013.12.007.

[29] D. Karaboga and B. Basturk, "Artificial Bee Colony (ABC) Optimization Algorithm for Solving Constrained Optimization Problems," in *Foundations of Fuzzy Logic and Soft Computing*, Berlin, Heidelberg, 2007, pp. 789–798, https://doi.org/10.1007/978-3-540-72950-1_77.

[30] A. Faramarzi, M. Heidarinejad, S. Mirjalili, and A. H. Gandomi, "Marine Predators Algorithm: A nature-inspired metaheuristic," *Expert Systems with Applications*, vol. 152, Aug. 2020, Art. no. 113377, https://doi.org/10.1016/j.eswa.2020.113377.

[31] S. Kaur, L. K. Awasthi, A. L. Sangal, and G. Dhiman, "Tunicate Swarm Algorithm: A new bio-inspired based metaheuristic paradigm for global optimization," *Engineering Applications of Artificial Intelligence*, vol. 90, Apr. 2020, Art. no. 103541, https://doi.org/10.1016/j.engappai.2020.103541.

[32] L. Abualigah, M. A. Elaziz, P. Sumari, Z. W. Geem, and A. H. Gandomi, "Reptile Search Algorithm (RSA): A nature-inspired meta-heuristic optimizer," *Expert Systems with Applications*, vol. 191, Apr. 2022, Art. no. 116158, https://doi.org/10.1016/j.eswa.2021.116158.

[33] B. Abdollahzadeh, F. S. Gharehchopogh, and S. Mirjalili, "African vultures optimization algorithm: A new nature-inspired metaheuristic algorithm for global optimization problems," *Computers & Industrial Engineering*, vol. 158, Aug. 2021, Art. no. 107408, https://doi.org/10.1016/j.cie.2021.107408.

[34] M. Braik, A. Hammouri, J. Atwan, M. A. Al-Betar, and M. A. Awadallah, "White Shark Optimizer: A novel bio-inspired meta-heuristic algorithm for global optimization problems," *Knowledge-Based Systems*, vol. 243, May 2022, Art. no. 108457, https://doi.org/10.1016/j.knosys.2022.108457.

[35] F. Wilcoxon, "Individual Comparisons by Ranking Methods," in *Breakthroughs in Statistics: Methodology and Distribution*, S. Kotz and N. L. Johnson, Eds. Springer, 1992, pp. 196–202.

[36] B. K. Kannan and S. N. Kramer, "An Augmented Lagrange Multiplier Based Method for Mixed Integer Discrete Continuous Optimization and Its Applications to Mechanical Design," *Journal of Mechanical Design*, vol. 116, no. 2, pp. 405–411, Jun. 1994, https://doi.org/10.1115/1.2919393.

[37] M. Pant, T. Radha, and V. P. Singh, "A Simple Diversity Guided Particle Swarm Optimization," in *2007 IEEE Congress on Evolutionary Computation*, Singapore, Sep. 2007, pp. 3294–3299, https://doi.org/10.1109/CEC.2007.4424896.

[38] X. Yao, Y. Liu, and G. Lin, "Evolutionary programming made faster," *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 2, pp. 82–102, Jul. 1999, https://doi.org/10.1109/4235.771163.