Buckling of Rectangular Plates with Embedded Stiffeners under Shear Stress

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ABSTRACT

This study analyzes the shear buckling behavior of rectangular plates with embedded stiffeners, employing the Finite Element Method (FEM) in ABAQUS. A total of 98 plate models were examined under shear stresses to assess the influence of key design parameters, including stiffener height, shape, location, and plate aspect ratio, on shear buckling resistance. The findings indicate that the optimal position for the embedded stiffener is at the center of the plate, irrespective of the configuration, resulting in an enhancement in shear resistance ranging from 14% to 32% compared to the unstiffened reference plates. Stiffeners positioned at one-third and one-quarter of the plate length yield more modest improvements, with shear resistance increases of 3% to 9% and 1% to 6%, respectively. For plates with an aspect ratio of 0.5, the optimal stiffener height was determined to be 30 mm, resulting in a 33% increase in shear resistance. Conversely, for plates with an aspect ratio of 1, the optimal stiffener height was found to be 40 mm, yielding a 76% increase in shear buckling resistance. The influence of the stiffener shape was also examined, with trapezoidal stiffeners demonstrating the most substantial enhancement (33% increase) for aspect ratio 0.5, and circular stiffeners exhibiting the most significant improvement (75% increase) for aspect ratio 1. The analysis further revealed that increasing the aspect ratio from 0.5 to 1 led to a substantial reduction in the shear buckling resistance, with a decrease of 64%. These findings underscore the crucial impact of stiffener parameters and aspect ratio on the shear buckling performance of stiffened plates, providing valuable guidance for the design and optimization of thin-walled structures subjected to shear loading.

Keywords-embedded stiffener; shear stresses; local buckling; Abaqus; steel plate

I. INTRODUCTION

In a multitude of structures, the presence of stiffened and unstiffened shell structures or shear plates is a common occurrence [1]. In the field of shipbuilding, thin-walled structural, aerospace, automobile, and construction sectors, the issue of a plate buckling under in-plane compressive and shear loading is of particular concern. The phenomenon of a plate losing its stability and buckling is well-documented in the engineering literature. This instability can lead to a sudden and severe failure, potentially resulting in the complete collapse of the structure [2]. The phenomenon of buckling can arise in circumstances involving compressive loading [3]. However, in the domain of solid mechanics, buckling is a multifaceted process that poses significant challenges, particularly for thin structures in areas subject to high compressive forces or stresses [4]. It is widely acknowledged that one of the primary factors contributing to the diminished load-bearing capacity of thin plates, which can lead to premature structural failure, is shear buckling. Consequently, the analysis of shear buckling in thin plates has garnered significant attention from researchers in recent decades. In their research, scholars employed a range of analytical, experimental, and numerical methods [5], leading to the accumulation of substantial data on the shear capability of steel plates. The buckling behavior of plates subjected to shear and edge compression was investigated in [6], where the impacts of thickness, slenderness ratio, and plate aspect ratio were studied numerically. The impact of boundary conditions and loadings was further investigated by examining various types of supports and loading. Finally, a comparison of the numerical and theoretical results was conducted. The current study focuses on the buckling behavior of plates, as well as the capabilities of the Plate-Buckling Program (PPB) and ABAQUS for performing linear and nonlinear buckling analysis. In addition, as noted in [7], the phenomenon of shear buckling was examined, with geometrical parameters, such as hole size, plate thickness, and hole type having been used. The numerical simulation findings concerning the critical buckling shear stress were validated by the theoretical analysis and mechanical experiment. The findings demonstrated a strong correlation between the numerical and experimental results. The study further revealed that for a wide range of hole types, the shear stress exhibited a comparable trend with the plate thickness. The study underscores the pivotal role of plate thickness and hole size in determining the shear stability of plates. When the hole size remains constant, the critical shear stress increases with an increasing plate thickness. Conversely, as hole size increases, the critical shear stress decreases. In addition to the work having mentioned the buckling of plates without stiffeners [8-10], several researchers have investigated the buckling of plates with stiffeners. Authors in [11-15] investigated the shear buckling behavior of longitudinally stiffened panels. The researchers tested four full-scale stiffened panels, exploring the impact of the position and stiffness of a trapezoidal longitudinal stiffener. The findings demonstrated a strong correlation between the finite element analysis and the experimental results. Authors in [16] studied the characterization of the elastic shear buckling of simply supported plates diagonally stiffened. Two extensive numerical parametric studies were conducted using the FEM for linear buckling analysis, encompassing a comprehensive practical range of critical parameters, such as the aspect ratio and stiffener mechanical characteristics. A review of the existing literature revealed a paucity of research on the buckling behavior of rectangular steel plates with embedded stiffeners being subjected to shear stresses. To address this research gap, several case studies were conducted using the Abaqus software [17], to investigate and better understand the previously unexplored properties of the steel plate buckling stability under shear stresses.

II. CHARACTERISTICS OF PLATES MODEL

The present study analyzes the stability of rectangular plate buckling with four simply supported edges and a variety of aspect ratios (*a*/*b*), stiffener shape, height, and location when subjected to shear stress through the usage of numerical simulation. To this end, the lengths of the plate along the *x*- and *y*-axes are denoted by the symbols *b* and *a*, respectively. Moreover, the plate possesses a uniform thickness (*t*) throughout the *z*-axis. Two cases of *a*/*b* were studied, the first was 0.5 and the second was 1, with the plate thickness having been maintained at 10 mm throughout. These were the referenced plates. The remaining plates were reinforced using embedded stiffeners, with the weight of the plate without stiffeners being equivalent to the weight with stiffeners, using a thickness equivalent to the original thickness (10 mm). Four shapes were utilized for the stiffeners (triangular, circular, rectangular, and trapezoidal), with heights of 10 mm, 20 mm, 30 mm, and 40 mm for each shape. In this paper, the center-line, quarter-lines, and the third line are examined. Furthermore, the plate models are designated such that the name can be used to identify the model's attributes. For instance, the name P1TQ10 shows that the plate with *a*/*b* 0.5 (P1), has a triangular stiffener (T) ten millimeters high and situated in quarter-line of the plate (Q). This is for triangular stiffeners, whereas for other plate models the abbreviation changes depend on the type of stiffener shape: P1RQ10 for rectangular stiffeners (shown in Figure 1), P1CR10 for circular stiffeners, and P1ZQ10 for trapezoidal stiffeners.

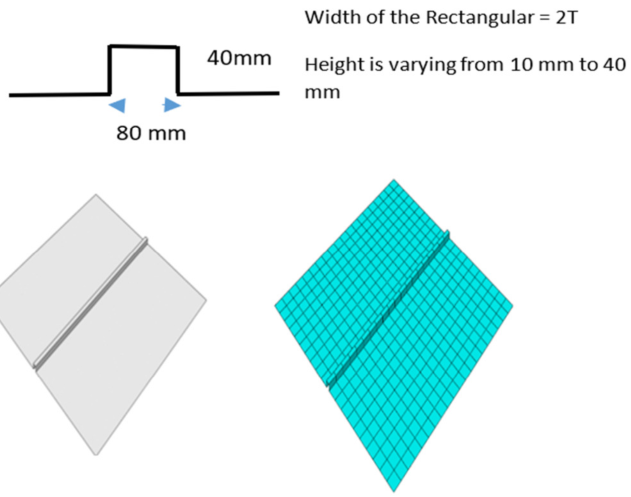

Fig. 1. 2D section and 3D Abaqus model of the plate with rectangular stiffener.

A. *Finite Element Modeling*

The steel plate model's FEM material attributes included a 200 GPa Young's modulus, a 250 MPa yield stress, a 0.3 Poisson's ratio, and a bulk density of 7850 kg/m3. For the linear buckling analysis, the eigenvalue approach was applied [18]. This computational method is deployed to determine the elastic structure's numerical buckling load. It is important to note that predicted buckling loads are often overestimated, as it is expected that the structure will behave linearly elastically. Consequently, the FEM is employed to identify the load that would induce structural divergence, should it be anticipated that the component would exhibit structural instability. The equilibrium equations for this study are derived by solving homogeneous algebraic equations with the eigenvector reflecting the primary buckling mode and the lowest eigenvalue corresponding to the buckling load [19]. The finite element program ABAQUS uses the sub-space numerical approach to solve the eigenvalue problem. In this research, the plates were represented by employing the S8R 6 element in a theoretical simulation using ABAQUS. This thin shell element possesses eight nodes, two curves, decreased integration points, and six degrees of freedom. At each node, three translational and three in-plane rotational degrees of freedom are available.

B. Loading and Boundary Conditions

The constraint of all plate models was achieved by using the displacement of the boundary condition to ensure the attainment of the correct solution. Each model was subjected to simple support from every edge. As presented in Figure 2, the plates were subjected to a uniform shear load of 1 N/mm^2. The load required to obtain the first buckling mode, i.e., the critical shear buckling load, was determined.

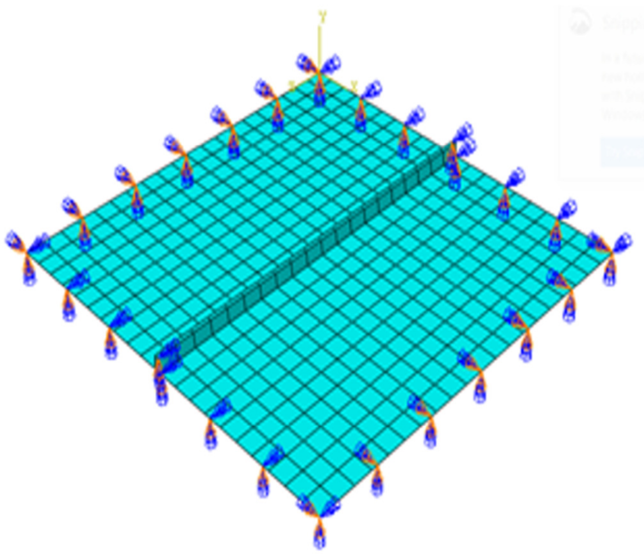

Fig. 2. Four edge simply supported rectangular plate.

C. Verification Study

The equation for plate buckling was utilized to ascertain the result for a rectangular plate measuring 1000 mm × 1000 mm × 1000 mm [20]. A comparison was conducted between the numerical result obtained from ABAQUS and the theoretical result derived from the equation. It was concluded that there was a substantial agreement between the numerical and theoretical results. Specifically, the theoretical result yielded 1688 kN, while ABAQUS yielded 1681 kN for the critical bulking load.

III. RESULTS AND DISCUSSION

A. Parametric Study

A series of parametric experiments were examined using the numerical application of ABAQUS to study the impact of several important parameters on the buckling stability of the steel plate under the influence of shear forces. The selected parameters encompass the aspect ratio (*a*/*b*), the aspect ratio (*b*/*t*), the configuration of stiffeners (shape, position, and stiffener-to-plate length ratio), and the effect of the critical shear bulging load and failure mode. The analysis incorporates the first mode of eigen buckling to provide a comprehensive demonstration of the effect of failure mode on the plate model.

B. Effect of Stiffeners Location

A study was conducted to investigate the impact of the stiffener location on shear buckling resistance. The experiment was performed in three locations: the first location was at the quarter, the second location was at the third, and the third location was in the middle of the plate. The effect of the stiffener's position on resistance to shear buckling was examined for all cases, aspect ratio, stiffener height-to-length ratios (*hs*/*b*), and shape, as illustrated in Figure 3. The findings demonstrated that the effective position for the embedded stiffener at the center of the plate was optimal for all models, as it prevented instability and maximized shear strength. Furthermore, the increase in shear resistance was from 14% to 32% due to the redistribution of stresses and the increase in section of moment of inertia, which reduced the slenderness ratio compared with the reference plates. Furthermore, an enhancement in the shear buckling resistance was observed in other samples when compared to the reference plates. However, stiffeners positioned in the third and quarter of the plate exhibited a reduction in shear buckling resistance compared to the stiffeners located in the central region of the plate. Specifically, the increase ratio for the third of the plate was 3% to 9%, and 1% to 6% in the quarter.

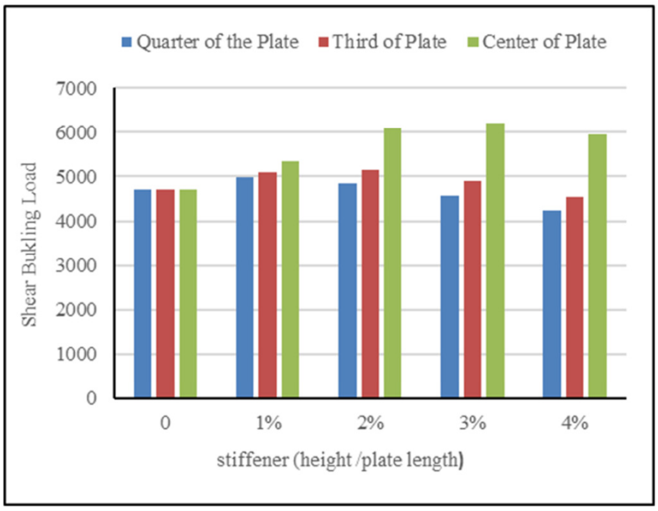

Fig. 3. Location effect for rectangular stiffener with aspect ratio 1.

C. Effect of Stiffeners Height

As presented in Figure 4, the impact of stiffener height has been examined by considering four *hs*/*b* ratios (1%, 2%, 3%, and 4%) with different stiffener forms, positions, and *a*/*b* ratios. The findings have been compared with the flat reference plates P1 and P2. The findings reveal that for an *a*/*b* ratio of 0.5, there is an increase in shear buckling resistance with an increase in *hs*/*b* ratio from 1% to 3%. Furthermore, an increase in *hs*/*b* ratio beyond 3% results in a reduction in shear buckling resistance. For an *a*/*b* ratio of 1, it is observed that the shear buckling resistance increases with an increase in *hs*/*b* ratio from 1% to 4%. Furthermore, the stiffener's most effective *hs*/*b* ratio is achieved when the stiffener's *hs*/*b* ratio is 3% for an *a*/*b* ratio equal 0.5, where the shear buckling resistance is 33% compared with the reference plate P1. Conversely, for an *a*/*b* ratio equal to 1, the shear buckling resistance reached a maximum value of 76%, as compared to the reference plate P2. This is attributable to the increase in the section moment of inertia, which is a primary factor in reducing the slenderness ratio. This, in turn, renders the plate more suitable and resistant to shear buckling loads.

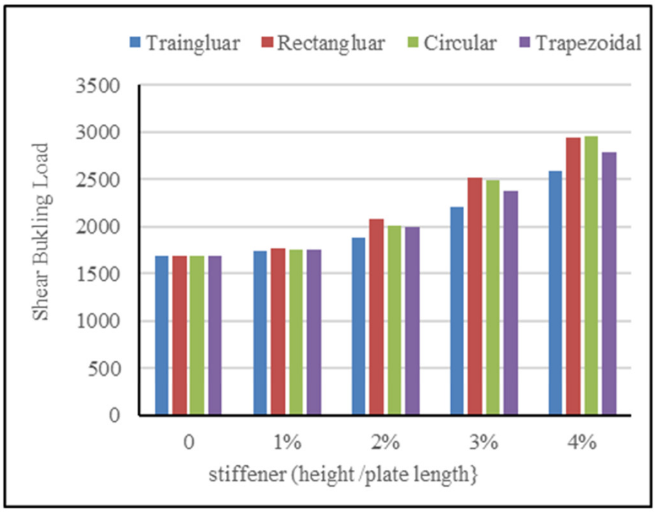

Fig. 4. High impact for different shapes of stiffener with a/b ratio equal to 1 at the center of the plate.

D. Effect of Stiffeners Shapes

In order to examine the effect of the stiffener shape on the shear bulking of plate, four shapes were studied: triangular, rectangular, circular, and trapezoidal. The shapes were examined with varying stiffener *hs*/*b* ratios, positions, and *a*/*b* ratios. The findings were then compared with the flat reference plates P1 and P2 (without stiffener), as shown in Figure 5. The findings indicate that all plate shapes exhibit enhanced resistance to shear buckling in comparison to the flat plates, attributable to the augmentation in section moment of inertia, which results in a reduction in the plates' slenderness ratio. Furthermore, for an *a*/*b* ratio of 0.5, the optimal configuration that enhanced the plate's stability and led to the maximum shear buckling resistance was observed to be a trapezoidal shape, which augmented the shear bulking load by 33% in comparison to alternative stiffener shapes. Conversely, for an a/b ratio of 1, the optimal configuration was identified as the circular shape, which increased the shear bulking load by 75% in comparison to other stiffener shapes.

E. Effect of Plate Aspect Ratio (a/b)

The impact of the plate aspect ratio *a*/*b* was studied by comparing two *a*/*b* ratios. The first ratio was set at 0.5, while the second was configured at 1, employing distinct stiffener shapes and positions. Additionally, the *hs*/*b* ratio was considered, and the results were compared with those of the flat plates P1 and P2. The findings indicated a substantial decrease in resistance to shear buckling with an increase in the aspect ratio, with a 64% reduction in moment of inertia when the aspect ratio increased from 0.5 to 1. This reduction in moment of inertia resulted in an enhancement of the slenderness ratio for the plate, as compared to the reference flat plates. Figure 6 displays the impact of aspect ratios *a*/*b* equal to 0.5 and 1, respectively, on the model samples P1RT20 and P2RT20 when compared to their flat plate specimens P1 and P2.

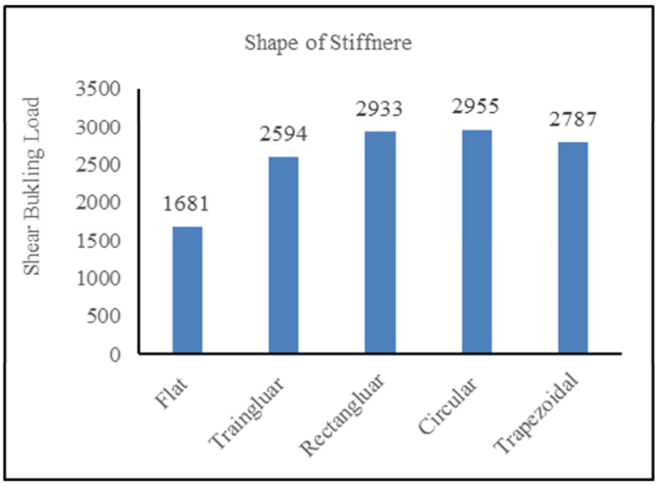

Fig. 5. Shape impact for circular stiffener with 1 aspect ratio.

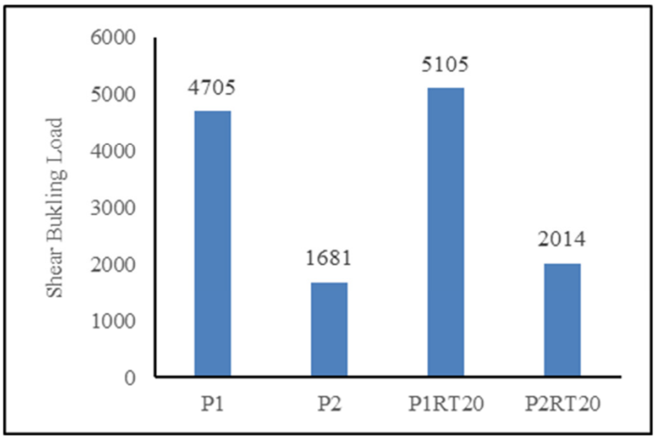

Fig. 6. Impact of aspect ratio a/b equal to 0.5 and 1 on the model samples.

F. Effect of Plate Aspect Ratio (b/t)

An experiment was conducted to ascertain the effect of the plate aspect ratio (*b*/*t*). The plate with *a*/*b* ratio 1 was selected, and ten ratios of *b*/*t* were considered, ranging from 20 to 200 with an interval of 20. Additionally, a variety of stiffener forms (triangular, rectangular, circular, trapezoidal) were examined, along with four stiffener *hs*/*b* ratios (1%, 2%, 3%, and 4%). As presented in Figures 7-10, the general behavior of the curves indicates that the shape of all stiffeners follows the same trend line until *b*/*t* reaches 100. Thereafter, the shear P1RT20, P2RT20, P1, and P2 buckling resistance begins to decrease with an increasing *b*/*t* up to 200. This is considered the maximum reduction due to the increasing slenderness ratio of the plate.

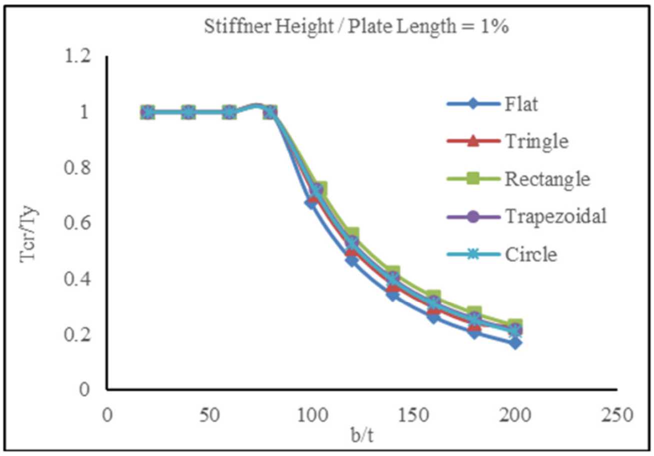

Fig. 7. Ratio between the elastic critical shear stress Tcr and the material yielding shear stress *Ty* to *b*/*t* ratio for stiffened plates with *hs*/*b* 1%.

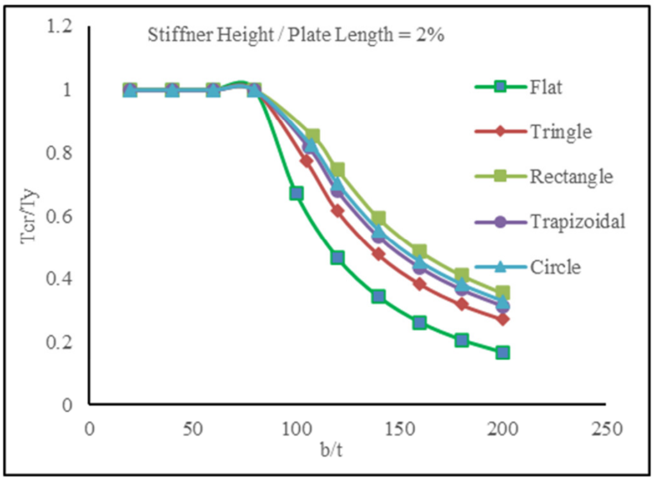

Fig. 8. Ratio between the elastic critical shear stress Tcr and the material yielding shear stress Ty to *b*/*t* ratio for stiffened plates with *hs*/*b* 2%.

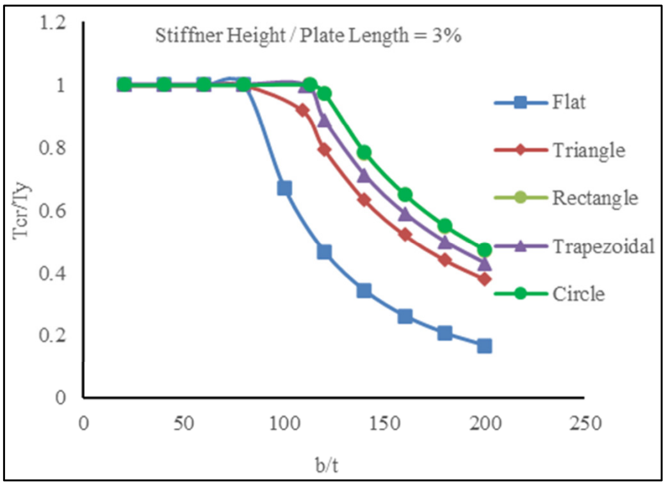

Fig. 9. Ratio between the elastic critical shear stress Tcr and the material yielding shear stress Ty to *b*/*t* ratio with *hs*/*b* 3%.

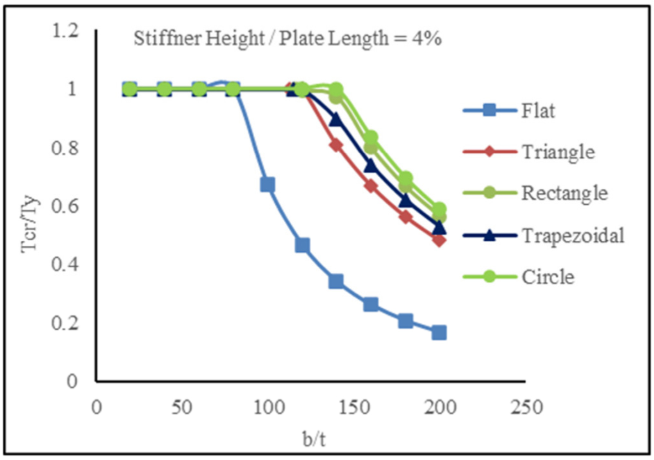

Fig. 10. Ratio between the elastic critical shear stress Tcr and the material yielding shear stress Ty to b/t ratio for stiffened plates with *hs*/*b* 4%.

Furthermore, as portrayed in Figure 7, for a *hs*/*b* ratio of 1%, the plate with stiffeners exhibited a modest augmentation in shear bulking load when compared to a flat plate. This disparity was more pronounced at the higher ratios of 2%, 3%, and 4%, as evidenced in Figures 8, 9, and 10, respectively. Furthermore, the findings presented in Figures 7 and 8 indicate that the plates with rectangular stiffeners exhibit optimal performance in terms of shear buckling resistance, particularly when the *hs*/*b* ratio is 1% and 2%. Conversely, the findings in Figures 9 and 10, with *hs*/*b* ratios of 3% and 4%, respectively, indicate that the plate with circular stiffeners is the most effective configuration.

IV. CONCLUSIONS

This study aimed to address a significant gap in the existing literature regarding the buckling behavior of rectangular steel plates with embedded stiffeners under shear stresses. While substantial research exists on the steel plate buckling, no prior work has systematically explored the influence of the embedded stiffeners on shear buckling resistance. The present research bridges this gap by employing the ABAQUS software to analyze a series of models, uncovering previously unknown properties of stiffened steel plates under shear stresses. The novelty of this study lies in its focus on determining the optimal stiffener configuration, shape, and position to enhance the shear buckling resistance, as well as its detailed exploration of how geometric parameters affect performance. The results reveal that the position, shape, and dimensions of stiffeners significantly impact the buckling resistance. Notably, placing the stiffener at the plate's center yielded the highest resistance improvements, with increases ranging from 14% to 32% compared to the reference flat plates. For an aspect ratio *a*/*b*=0.5 the optimal *hs*/*b* stiffener ratio of 3% increased resistance by 33% relative to a reference plate, while for *a*/*b*=1 a ratio of 4% achieved a 76% increase. Furthermore, the configuration of the stiffener played a critical role; trapezoidal stiffeners were most effective for *a*/*b*=0.5, whereas circular stiffeners performed best for *a*/*b*=1 achieving resistance improvements of 33% and 75%, respectively. This work also highlights a general tendency: increasing the aspect ratio *a*/*b* considerably reduces the shear buckling resistance, with a 64%

reduction having been observed as *a*/*b* increased from 0.5 to 1. Similarly, an increase in the *b*/*t* ratio negatively impacted the resistance across all models. In terms of stiffener effectiveness, rectangular stiffeners were optimal for *hs*/*b* ratios ranging from 1% to 2%, while circular stiffeners exhibited superior performance for ratios between 3% and 4%. In comparison to previous studies, which have predominantly concentrated on flat plates or unstiffened configurations, this research offers a novel perspective by demonstrating how stiffener properties and placement can be optimized to significantly enhance the buckling stability. The findings offer valuable insights for structural engineering, enabling the design of more efficient steel plates in applications where shear buckling resistance is critical.

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